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AN EMBEDDED CRACK IN A LAMINATE WITH ADHESIVE LAYERS.

by

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~~and~~

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The five-layered composite as treated in this work consists of three plates bonded together with adhesives. The thickness of the adhesives, though small, is included in the analysis resulting in the five layer geometry. A dominant flaw is assumed to prevail at the center of the middle layer when the composite is being stretched uniformly and to overshadow the other types of mechanical damages such as matrix/fiber debonding, fiber breaking, etc.

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In order to gain a knowledge of the failure mode of the layered composite, the load transmission characteristics which depend upon the geometrical and material constants must be known through a stress analysis. This is accomplished by appealing to an approximate theory of laminates developed from variational principles. The character of the solution, particularly that portion around the crack front, is three-dimensional and is sufficiently accurate for predicting physically realizable failure modes. In addition to analyze the influence of the adhesive layers, the stress distribution obtained ~~here~~ may be further applied in conjunction with a failure criterion to determine whether delamination will occur prior to unstable crack propagation or not. The pertinent parameter, stress intensity factor, is obtained for a variety of material and geometrical constants.

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SUMMARY

The five-layered composite as treated in this work consists of three plates bonded together with adhesives. The thickness of the adhesives, though small, is included in the analysis resulting in the five layer geometry. A dominant flaw is assumed to prevail at the center of the middle layer when the composite is being stretched uniformly and to overshadow the other types of mechanical damages such as matrix/fiber debonding, fiber breaking, etc.

In order to gain a knowledge of the failure mode of the layered composite, the load transmission characteristics which depend upon the geometrical and material constants must be known through a stress analysis. This is accomplished by appealing to an approximate theory of laminates developed from variational principles. The character of the solution, particularly that portion around the crack front, is three-dimensional and is sufficiently accurate for predicting physically realizable failure modes. In addition to analyze the influence of the adhesive layers, the stress distribution obtained here may be further applied in conjunction with a failure criterion to determine whether delamination will occur prior to unstable crack propagation or not. The pertinent parameter, stress intensity factor, is obtained for a variety of material and geometrical constants.

INTRODUCTION

Advanced laminate composites are finding increasing applications in load carrying aircraft components. As a result, there has been a growing need for reliable methods of determining strength and/or life expectancy of composites. The fracture mechanics approach addresses the physical processes of material damage that actually occur in the composite. Analytical modeling of the variety of failures in composites such as matrix/fiber debonding, fiber breaking, matrix cracking, ply delamination, free edge effects, etc., if carried out indiscriminately, yields little or no useful information. The other extreme measure of simply testing specimen after specimen is no better as very little understanding could be gained.

For those laminates such as graphite/epoxy plates that exhibit brittle fracture behavior, there prevails a distinct point of instability on the load-deformation curve; it is possible to identify the physical damage such as a dominant crack in the composite just prior to load instability. Attention should then be focused in determining the size, shape and location of this dominant crack. The other damages such as fiber breaking, matrix/fiber debonding, etc., that occurred in the early part of load history should be considered as dissipated energy that is no longer available at the final stage of load instability. This is a very brief description of the idea on which fracture mechanics is based upon.

In a previous report [1], the authors developed a method for solving the stresses in a multi-layered composite plate containing a through-the-thickness crack at the center. A three-layered laminate was solved numerically as an illustrative example for the crack front stress intensity factors. The initial assumption of a crack cutting through all the layers, however, may not be realistic. This is because the zone of hydrostatic tension or high dilatation occurs only at the center of the middle layer of the laminate. In fact, the outer layers distort relatively more and are less prone to fail by cracking through the thickness coinciding with that in the middle layer. There is the possibility that interface or interlayer delamination accompanied by crack propagation in the middle layer may occur first before the outerlayers begin to separate*. The load transmission characteristics in a layered composite can be very complex and will not be understood unless effective stress analysis is performed and used jointly with a realistic fracture criterion.

As a first prerequisite, this report attempts to model the laminate with a center crack in the middle layer while the outerlayers are uncracked. This obviously makes the problem considerably more difficult. In addition, adhesive layers

*This has been observed in a three-layered plexiglass plate that was pulled apart with a crack in the middle layer. The middle layer can be separated completely by crack propagation with the outerlayers still intact. This test was performed at the Lehigh University Institute of Fracture and Solid Mechanics.

are inserted between the adjoining layers. Hence, a three-layered plate will, in fact, contain five layers if the two adhesive layers are also included. Whether the adhesive layers will or will not have a significant influence on delamination and/or crack propagation can only be answered by appealing to a stress and fracture analysis.

An approximate theory of laminates similar to that given in [1] is used. The governing equations are derived from the complementary potential energy theorem in variational calculus. Near the crack front, the approximate approach does preserve the qualitative character of the solution which is inherently three-dimensional [2]. What is different is the load transmission characteristics that are reflected through the stress intensity parameter. A numerical scheme is developed for finding these parameters for various geometrical and material constants. The results are reported herein. The stress components around the crack are also calculated and presented which are needed in the study of delamination.

AN APPROXIMATE THREE DIMENSIONAL THEORY FOR LAMINATES

The principle of minimum complementary energy as used in calculus of variations is a standard procedure for developing approximate theories of plates and has been widely applied in the field of plates and shells. An approximate form of the solution, say in terms of the stress components, is first assumed which are then substituted into the complementary potential energy known as a functional. Minimization of this functional with respect to the admissible variations of the stress components leads to a best approximation of the exact solution in a weighted sense.

The procedure to be employed for laminated plates is to assume that each of the stress components in the pth layer can be approximated as a product of a function of the transverse variable z multiplied by a function of the in-plane variables x and y, i.e.,

$$[\sigma_x, \sigma_y, \tau_{xy}]_p = f''_p(z)[S_x(x,y),\ S_y(x,y),\ S_{xy}(x,y)]_p$$

$$[\tau_{xz}, \tau_{yz}]_p = -\ f'_p(z)[Z_x(x,y),\ Z_y(x,y)]_p \tag{1}$$

$$[\sigma_z]_p = f_p(z)[Z_z(x,y)]_p$$

in which prime designates differentiation with respect to the variable z. The function $f_p(z)$ describes the stress variations in the thickness direction through the pth layer of the laminate.

This assumed form of the stress field is substituted into the complementary potential energy functional. Minimization of the functional with respect to f(z), S_x, S_y, S_{xy}, Z_x, Z_y and Z_z of each layer is then carried out whereby the conditions of equilibrium are enforced. The choice of the stress and displacement expressions should be such that the approximate solution satisfies the continuity conditions across the material interfaces as closely as possible. For example, the traction continuity conditions are satisfied by taking

$$[Z_x, Z_y, Z_z]_p = [Z_x, Z_y, Z_z] \tag{2}$$

and

$$f_p(z) = f_{p+1}(z), \; f'_p(z) = f'_{p+1}(z) \tag{3}$$

in which z corresponds to the respective values at the interface between layers p and p+1. This assumption precludes an exact evaluation of the state of affairs in those regions of the outerlayers that are close to the crack. Nevertheless, the solution is expected to be accurate at distances sufficiently far away from the crack. Moreover, the qualitative features of the crack front stresses will not be altered by assumptions (2) and (3). The same argument leads to the following displacement continuity conditions:

$$[S_x, S_y, S_{xy}]_p = [S_x, S_y, S_{xy}] \tag{4}$$

Finally, the assumed stress field takes the form

$$[\sigma_x, \sigma_y, \tau_{xy}]_p = f_p''(z)[S_x, S_y, S_{xy}]$$

$$[\tau_{xz}, \tau_{yz}]_p = - f_p'(z)[Z_x, Z_y] \qquad (5)$$

$$[\sigma_z]_p = f_p(z) Z_z$$

The stress equilibrium conditions in three-dimensional elasticity when enforced on the stresses σ_x, σ_y,---, etc., in equations (5) yield a set of equations involving the variables x and y only:

$$Z_x = \frac{\partial S_x}{\partial x} + \frac{\partial S_{xy}}{\partial y}$$

$$Z_y = \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_y}{\partial y} \qquad (6)$$

$$Z_z = \frac{\partial Z_x}{\partial x} + \frac{\partial Z_y}{\partial y}$$

Note that once S_x, S_y and S_{xy} are known, the quantities Z_x, Z_y and Z_z that govern the transverse stress distribution can be obtained by means of equations (6). This simplification is a direct consequence of the product form of solution assumed in equations (1).

The Variational Principle of Minimum Complementary Potential Energy for Laminates. The theorem of minimum complementary potential energy in variational calculus may be used to develop

an approximate theory for laminated plates. This approach was used in [1] and the same procedure is applicable to the current system. Without going into the details [1], the differential equations governing the stresses S_x, S_y,---, Z_y and the weighted displacements u_x, u_y and u_z can be written as

$$
\begin{aligned}
& I_1 S_x + I_2 S_y + I_4 Z_z + \frac{\partial u_x}{\partial x} = 0 \\
& I_1 S_y + I_2 S_x + I_4 Z_z + \frac{\partial u_y}{\partial y} = 0 \\
& 2(I_1 - I_2) S_{xy} + \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = 0 \\
& I_3 Z_x + u_x + \frac{\partial u_z}{\partial x} = 0 \\
& I_3 Z_y + u_y + \frac{\partial u_z}{\partial y} = 0 \\
& I_4 (S_x + S_y) + I_5 Z_z + u_z = 0
\end{aligned}
\tag{7}
$$

in which the following contractions have been made:

$$
\begin{aligned}
& I_1 = \frac{1}{E_1} \int_{-h_1}^{h_1} [f_1''(z)]^2 dz, \quad I_2 = -\nu_1 I_1 \\
& I_3 = \frac{2(1+\nu_1)}{E_1} \int_{-h_1}^{h_1} [f_1'(z)]^2 dz, \quad I_4 = -\frac{\nu_1}{E_1} \int_{-h_1}^{h_1} f_1(z) f_1''(z) dz \\
& I_5 = \frac{1}{E_1} \int_{-h_1}^{h_1} [f_1(z)]^2 dz
\end{aligned}
\tag{8}
$$

In equations (7), the weighted displacement components u_x, u_y and u_z are related to the actual displacements $\bar{U}_x$, $\bar{U}_y$ and $\bar{U}_z$ through

$$u_x = - \int_{H_p} \bar{U}_x f_p''(z)dz$$

$$u_y = - \int_{H_p} \bar{U}_y f_p''(z)dz$$

$$u_z = \int_{H_p} \bar{U}_z f_p'(z)dz$$

where H_p denotes the thickness of the pth layer.

Equations (6) may be used to reduce the system of equations (7) in terms of the unknowns Z_x, Z_y and u_z. The results are

$$Z_x - a_6\nabla^2 Z_x - \frac{\partial}{\partial x}(a_1\nabla^2 u_z + a_2 u_z) = 0$$

$$Z_y - a_6\nabla^2 Z_y - \frac{\partial}{\partial y}(a_1\nabla^2 u_z + a_2 u_z) = 0$$

$$a_3\nabla^4 u_z + a_4\nabla^2 u_z + a_5 u_z = 0 \tag{9}$$

$$\frac{\partial Z_x}{\partial x} + \frac{\partial Z_y}{\partial y} + a_7\nabla^2 u_z + a_8 u_z = 0$$

The symbols ∇^2 and ∇^4 represent the harmonic and biharmonic operators in the variables x and y while a_1, a_2,---, a_8 are complicated functions of I_1, I_2,---, I_5 as defined in equations (8), i.e.,

$$a_1 = \frac{I_4(I_3-2I_4)+2I_1I_5}{2(I_1-I_2)[I_4(I_3-2I_4)+I_5(I_1+I_2)]}$$

$$a_2 = \frac{2I_4-I_3}{2[I_4(I_3-2I_4)+I_5(I_1+I_2)]}$$

$$a_3 = I_4^2 - I_1I_5$$

$$a_4 = - 2I_4(I_1-I_2) + I_1I_3 \tag{10}$$

$$a_5 = \frac{I_3}{2(I_1-I_2)}, \quad a_6 = \frac{I_3}{2(I_1-I_2)}$$

$$a_7 = \frac{I_4}{I_4(I_3-2I_4)+I_5(I_1+I_2)}$$

$$a_8 = \frac{I_1+I_2}{I_4(I_3-2I_4)+I_5(I_1+I_2)}$$

Once Z_x, Z_y and u_z are found, the remaining unknowns u_x, u_y, etc., can be obtained in a straightforward fashion since

$$u_x = - I_3Z_x - \frac{\partial u_z}{x}$$

$$u_y = - I_3Z_y - \frac{\partial u_z}{\partial y} \tag{11}$$

and

$$S_x = \frac{1}{I_1^2-I_2^2}\ \{I_1 \frac{\partial^2 u_z}{\partial x^2} - I_2 \frac{\partial^2 u_z}{\partial y^2} + [I_1 I_3 - I_4(I_1-I_2)] \frac{\partial Z_x}{\partial x}$$

$$- [I_2 I_3 + I_4(I_1-I_2)] \frac{\partial Z_y}{\partial y}\}$$

$$S_y = \frac{1}{I_1^2-I_2^2}\ \{I_1 \frac{\partial^2 u_z}{\partial y^2} - I_2 \frac{\partial^2 u_z}{\partial x^2} + [I_1 I_3 - I_4(I_1-I_2)] \frac{\partial Z_y}{\partial y}$$

$$- [I_2 I_3 + I_4(I_1-I_2)] \frac{\partial Z_x}{\partial x}\} \tag{12}$$

$$S_{xy} = \frac{1}{I_1-I_2} \frac{\partial^2 u_z}{\partial x \partial y} + \frac{I_3}{2(I_1-I_2)} \left(\frac{\partial Z_x}{\partial y} + \frac{\partial Z_y}{\partial x}\right)$$

$$Z_z = \frac{\partial Z_x}{\partial x} + \frac{\partial Z_y}{\partial y}$$

This completes the basic formulation of the laminate theory. As it can be seen from the equations just developed, the results involve numerous geometric and material parameters.

A FIVE-LAYER LAMINATE WITH A CRACK IN THE CENTER LAYER

Consider a symmetrically layered composite plate which contains a crack of length 2a through the middle layer. The five-layer plate is assumed to be made of three different elastic materials with the stacking sequence as that shown in Figure 1. The subscripts 1, 2 and 3 attached to the Young's modulus E, Poisson's ratio ν and layer thickness h refer to those properties belonging to the individual layer. In Figure 1, the crack length 2a is taken to be small in comparison with the in-plane dimensions of the plate such that the boundary effect on the stress field can be neglected.

A remote tensile stress of magnitude σ_o applied to the laminate. By the application of the principle of superposition, the equivalent problem to be solved here is that of normal traction applied to the surface of the crack. This traction can be estimated from the same laminate without the crack as [3]:

$$\sigma_y = \frac{\left[\sum_{n=1}^{3}(h_n/h_1)\right]\left\{\sum_{m=1}^{3}\left[\left(\frac{E_m}{E_1}\right)\left(\frac{1-\nu_1\nu_m}{1-\nu_m^2}\right)\right]\right\}}{\left\{\sum_{n=1}^{3}\left[\left(\frac{E_n}{E_1}\right)^2\left(\frac{h_n}{h_1}\right)^2\left(\frac{1-\nu_1^2}{1-\nu_n^2}\right)\right]\right\}+D}\,\sigma_o = K\sigma_o \qquad (13)$$

in which

$$D = 2\left(\frac{E_2}{E_1}\right)\left(\frac{h_2}{h_1}\right)\left(\frac{1-\nu_1\nu_2}{1-\nu_2^2}\right) + 2\left(\frac{E_3}{E_1}\right)\left(\frac{h_3}{h_1}\right)\left(\frac{1-\nu_1\nu_3}{1-\nu_3^2}\right) + $$
$$+ 2\left(\frac{E_2}{E_1}\right)\left(\frac{E_3}{E_1}\right)\left(\frac{h_2}{h_1}\right)\left(\frac{h_3}{h_1}\right)\frac{(1-\nu_1^2)(1-\nu_2\nu_3)}{(1-\nu_2^2)(1-\nu_3^2)} \qquad (14)$$

Due to symmetry, the differential equations (9) are to be solved subjected to the following conditions:

$$\tau_{xy}(x,o,z) = \tau_{yz}(x,0,z) = 0$$

$$\sigma_y(x,o,z) = - K\sigma_o f_1''(z), \quad 0\leq x<a$$

$$\bar{U}_y(x,o,z) = 0, \quad x\geq a \tag{15}$$

$$\tau_{xy}(o,y,z) = \tau_{xz}(o,y,z) = 0; \quad \bar{U}_x(o,y,z) = 0$$

$$\tau_{xy}(x,y,o) = \tau_{yz}(x,y,o) = 0; \quad \bar{U}_z(x,y,o) = 0$$

Eliminating the z-dependence in equations (15) renders

$$S_{xy}(x,0) = Z_y(x,o) = 0$$

$$S_y(x,o) = - Kp_o, \quad 0\leq x<a \tag{16}$$

$$u_y(x,o) = 0, \quad x\geq a$$

$$S_{xy}(o,y) = Z_x(o,y) = u_x(o,y) = 0$$

In the absence of out-of-plane bending, the last of equations (15) is satisfied when the functions $f_p(z)$ are required to be symmetric about the mid-plane z=0. In equations (16), the equivalent stress p_o is introduced such that $p_o = \sigma_o h_1{}^2$.

Solution Procedure. Fourier transform techniques will be used to solve the system of differential equations (9). Depending on the evenness and oddness of the function with respect to its argument, either the Fourier cosine or sine transform may be employed. For example, the function $S_x{}^c(x,y)$, $S_{xy}^s(x,y)$ will denote, respectively, the Fourier cosine and sine transform of the functions $S_x(x,y)$ and $S_{xy}(x,y)$. The same notation applied to the other functions $S_y{}^c(x,y)$, $Z_x{}^s(x,y)$, etc.' i.e.,

$$S_x(x,y) = \frac{2}{\pi}\int_0^\infty S_x{}^c(s,y)\cos(sx)ds$$

$$S_y(x,y) = \frac{2}{\pi}\int_0^\infty S_y{}^c(s,y)\cos(xs)ds$$

$$S_{xy}(x,y) = \frac{2}{\pi}\int_0^\infty S_{xy}{}^s(s,y)\sin(sx)ds \tag{17}$$

$$Z_x(x,y) = \frac{2}{\pi}\int_0^\infty Z_x{}^s(s,y)\sin(sx)ds$$

$$Z_y(x,y) = \frac{2}{\pi}\int_0^\infty Z_y{}^c(s,y)\cos(sx)ds$$

$$Z_z(x,y) = \frac{2}{\pi}\int_0^\infty Z_z{}^c(x,y)\cos(sx)ds$$

and the same applied to the displacements

$$u_x(x,y) = \frac{2}{\pi}\int_0^\infty u_x{}^s(s,y)\sin(sx)ds$$

$$u_y(x,y) = \frac{2}{\pi}\int_0^\infty u_y{}^c(s,y)\cos(sx)ds \tag{18}$$

$$u_z(x,y) = \frac{2}{\pi}\int_0^\infty u_z{}^c(s,y)\cos(sx)ds$$

Making use of equations (17) and (18) and without going into details, a solution may be written as

$$S_x{}^c(s,y) = \frac{I_3}{I_1-I_2} qB(s)e^{-qy} + \frac{1}{I_1^2-I_2^2} \mathrm{Re}\{[I_3(sI_1P_1$$

$$+ \rho I_2P_2) - I_4(I_1-I_2)(sP_1-\rho P_2)]e^{-\rho y}\}$$

$$- \frac{2}{I_1^2-I_2^2} \mathrm{Re}[(I_1s^2 + I_2\rho^2)A_1e^{-\rho y}]$$

$$S_y{}^c(s,y) = - \frac{I_3}{I_1-I_2} qB(s)e^{-qy} - \frac{1}{I_1^2-I_2^2} \mathrm{Re}\{[I_3(sI_2P_1$$

$$+\rho I_1P_2) + I_4(I_1-I_2)(sP_1 - \rho P_2)]e^{-\rho y}\}$$

$$+ \frac{2}{I_1^2-I_2^2} \mathrm{Re}[(I_1\rho^2 + I_2s^2)A_1e^{-\rho y}]$$

$$S_{xy}{}^s(x,y) = \frac{2}{I_1-I_2} \mathrm{Re}[s\rho A_1e^{-\rho y}] - \frac{I_3}{2(I_1-I_2)}\{\frac{q^2+s^2}{s} B(s)e^{-qy}$$

$$+ \mathrm{Re}[(\rho P_1 + sP_2)e^{-\rho y}]\} \qquad (19)$$

$$Z_xs(s,y) = \frac{q}{s} B(s)e^{-qy} + \mathrm{Re}[P_1e^{-\rho y}]$$

$$Z_y{}^s(s,y) = B(s)e^{-qy} + \mathrm{Re}[P_2e^{-\rho y}]$$

$$Z_z{}^c(s,y) = \mathrm{Re}[(sP_1 - \rho P_2)e^{-\rho y}]$$

$$u_x{}^s(s,y) = \mathrm{Re}[(2sA_1 - I_3P_1)e^{-\rho y}] - I_3 \frac{q}{s} B(s)e^{-qy}$$

$$u_y{}^c(s,y) = \mathrm{Re}[(2\rho A_1 - I_3P_2)e^{-\rho y}] - I_3 B(s)e^{-qy}$$

$$u_z{}^c(s,y) = 2 \mathrm{Re}[A_1e^{-\rho y}]$$

in which Re denotes the real part of a complex function and $A_1(s)$ and $B(s)$ are the unknowns to be determined from the boundary conditions. In equations (19), the quantities ρ, q, P_1 and P_2 represent

$$\rho = [s^2 - \frac{a_4}{2a_3} + i\sqrt{\frac{a_5}{a_3} - (\frac{a_4}{2a_3})^2}]^{1/2} = (s^2 - \gamma_1 + i\gamma_2)^{1/2}$$

$$= \rho_R + i\rho_I$$

$$q = (s^2 + \frac{1}{a_6})^{1/2} \tag{20}$$

$$P_1 = 2s \frac{a_1(s^2-\rho^2)-a_2}{a_6(s^2-\rho^2)+1} A_1$$

$$P_2 = 2\rho \frac{a_1(s^2-\rho^2)-a_2}{a_6(s^2-\rho^2)+1} A_1$$

Enforcing the boundary conditions (16) yields

$$B(s) = - \mathrm{Re}[2\rho \frac{a_1(s^2-\rho^2)-a_2}{a_6(s^2-\rho^2)+1} A_1(s)]$$

$$I_m[A_1(s)] = \beta(s) \, \mathrm{Re}[A_1(s)] \tag{21}$$

as well as a set of dual integral equations

$$\frac{2}{\pi} \int_0^\infty \frac{1}{s} A(s)\cos(sx)ds = 0, \quad x \geq a$$

$$\frac{2}{\pi} \int_0^\infty g(s) \, A(s)\cos(sx)ds = - Kp_o, \quad 0 \leq x < a \tag{22}$$

governing the unknown $A(s)$ where

$$A(s) = sg_1(s) \, \mathrm{Re}[A_1(s)] \tag{23}$$

In equations (21), (22) and (23) the following contractions have been made:

$$\beta(s) = -\left[\rho_R\left(s^2 + \frac{I_3}{2a_6}\delta_1\right) + \rho_I \frac{I_3}{2a_6}\delta_2\right] \Big/ \left[\rho_R \frac{I_3}{2a_6}\delta_2 - \rho_I\left(s^2 + \frac{I_3}{2a_6}\delta_1\right)\right]$$

$$\delta_1 = \frac{(a_1\gamma_1 - a_2)(a_6\gamma_1 + 1) + a_1 a_6 \gamma_2^2}{(a_6\gamma_1+1)^2 + a_6^2\gamma_2^2}$$

$$\delta_2 = \frac{\gamma_2(a_1 + a_2 a_6)}{(a_6\gamma_1+1)^2 + a_6^2\gamma_2^2}$$

$$g_1(s) = 2\,\mathrm{Re}[\rho(1+i\beta)] \tag{24}$$

$$g_2(s) = \frac{2}{I_1 - I_2}\,\mathrm{Re}\Big\{(1+i\beta)\Big[(I_1\rho^2 + I_2 s^2) + \frac{a_1(s^2-\rho^2) - a_2}{a_6(s^2-\rho^2)+1}$$

$$\left[I_3(I_1+I_2)q\rho - I_3(I_2 s^2 + I_1\rho^2) - I_4(I_1-I_2)(s^2-\rho^2)\right]\Big]\Big\}$$

$$g(s) = \frac{g_2(s)}{s g_1(s)}$$

Following [1], the solution to equations (22) may be expressed in terms of a new function $\Phi(\xi)$ as

$$A(s) = \frac{K p_o a}{2\kappa}\, s \int_0^1 \sqrt{\xi}\,\Phi(\xi)\, J_o(sa\xi)\,d\xi \tag{25}$$

where

$$\kappa = \frac{1}{\left(1 - \frac{\gamma_2}{2}\delta_3\right)(I_1 - I_2)}\Big\{(I_1 + I_2) + \delta_3\Big[-I_1\gamma_2 + (I_1 - I_2)\left(\frac{I_3}{2} - I_4\right)$$

$$(\gamma_1\delta_2 + \gamma_2\delta_1) - \delta_2\frac{I_3}{2a_6}(I_1+I_2)\Big]\Big\} \tag{26}$$

and

$$\delta_3 = (\frac{\gamma_2}{2} - \frac{I_3}{2a_6}\,\delta_2) \tag{27}$$

The function $\Phi(\xi)$ can be computed from a Fredholm integral equation of the second kind:

$$\Phi(\xi) + \int_0^1 \Phi(t)\, L(\xi,t)dt = -\sqrt{\xi} \tag{28}$$

The symmetric kernel $L(\xi,t)$ is defined by

$$L(\xi,t) = \frac{\sqrt{\xi t}}{\kappa}\int_0^\infty u[g(\frac{u}{a}) - \kappa]\, J_o(u\xi)J_o(ut)du \tag{29}$$

where J_o is the zero order Bessell function of the first kind.

Making use of equations (19), (21) and (23) the stresses in equations (17) take the form

$$S_x(x,y) = \frac{2Kp_oa^2}{\kappa}\int_0^\infty \frac{\psi(s)}{g_1(s)}\, \mathrm{Re}\{(I_1s^2 + I_2\rho^2)\frac{(1+i\beta)}{I_1^2-I_2^2}[(\delta_1-i\delta_2)I_3-1]e^{-\rho y}$$

$$- (\delta_1-i\delta_2)(1+i\beta)(\gamma_1 - i\gamma_2)\,\frac{I_4}{I_1+I_2}e^{-\rho y}$$

$$- (\delta_1-i\delta_2)(1+i\beta)\,\frac{q\rho I_3}{I_1-I_2}e^{-qy}\}\cos(sx)ds$$

$$S_y(x,y) = \frac{2Kp_oa^2}{\kappa}\int_0^\infty \frac{\psi(s)}{g_1(s)}\, \mathrm{Re}\{(I_2s^2 + I_1\rho^2)\frac{(1+i\beta)}{I_1^2-I_2^2}[1-(\delta_1-i\delta_2)I_3]e^{-\rho y}$$

$$- (\delta_1-i\delta_2)(1+i\beta)(\gamma_1-i\gamma_2)\,\frac{I_4}{I_1+I_2}e^{-\rho y}$$

$$+ (\delta_1-i\delta_2)(1+i\beta)\,\frac{q\rho I_3}{I_1-I_2}e^{-qy}\}\cos(sx)ds$$

$$S_{xy}(x,y) = \frac{2Kp_o a^2}{\kappa}\int_0^\infty \frac{\psi(s)}{g_1(s)}\,\mathrm{Re}\{\frac{\rho s}{I_1-I_2}\,[(1+i\beta)<1-(\delta_1-i\delta_2)I_3>e^{-\rho y}$$

$$+\ (\delta_1-i\delta_2)(1+i\beta)\,\frac{I_3(s^2+q^2)}{2s^2}e^{-qy}]\}\sin(sx)ds \qquad (30)$$

$$Z_x(x,y) = \frac{2Kp_o a^2}{\kappa}\int_0^\infty \frac{\psi(s)}{g_1(s)}\,\mathrm{Re}\{(\delta_1-i\delta_2)(1+i\beta)(se^{-\rho y}$$

$$-\ \frac{q\rho}{s}e^{-qy})\}\sin(sx)ds$$

$$Z_y(x,y) = \frac{2Kp_o a^2}{\kappa}\int_0^\infty \frac{\psi(s)}{g_1(s)}\,\mathrm{Re}\{(\delta_1-i\delta_2)(1+i\beta)\rho(e^{-\rho y}-e^{-qy})\}$$

$$\cos(sx)ds$$

$$Z_z(x,y) = \frac{2Kp_o a^2}{\kappa}\int_0^\infty \frac{\psi(s)}{g_1(s)}\,\mathrm{Re}\{(\delta_1-i\delta_2)(1+i\beta)(\gamma_1-i\gamma_2)e^{-\rho y}\}$$

$$\cos(sx)ds$$

in which

$$\psi(s) = \int_0^1 \sqrt{\xi}\ \Phi(\xi)J_o(sa\xi)d\xi \qquad (31)$$

Equations (30) in conjunction with the thickness functions $f_p(z)$ determine the stress distribution in the laminated plate.

Three-Dimensional Crack Front Stress Field. The singular behavior of the quantities S_x, S_y, etc., can be obtained by application of equations (30) in which only the terms that contribute to the divergence of the improper integrals at $x=\pm a$ and $y=0$ are retained. Expanding the integrals in

equations (30) for large s and multiplying the stresses by the functions $f_1(z)$, $f_1'(z)$ and $f_1''(z)$ according to equations (1), the following crack front stress distribution is obtained:

$$\sigma_x = K\Phi(1)\,\frac{\sigma_p(z)a^{1/2}}{(2r)^{1/2}}\left[\cos\frac{\theta}{2} - \frac{1}{2}\sin\frac{3\theta}{2}\right] + O(r^o)$$

$$\sigma_y = K\Phi(1)\,\frac{\sigma_p(z)a^{1/2}}{(2r)^{1/2}}\left[\cos\frac{\theta}{2} + \frac{1}{2}\sin\frac{3\theta}{2}\right] + O(r^o)$$

$$\tau_{xy} = K\Phi(1)\,\frac{\sigma_p(z)a^{1/2}}{(2r)^{1/2}}\,\frac{1}{2}\sin\theta\,\cos\frac{3\theta}{2} + o(r^o) \qquad (32)$$

$$\sigma_z = K\Phi(1)\,\frac{\sigma_T(z)a^{1/2}}{(2r)^{1/2}}\cos\frac{\theta}{2} + O(r^o)$$

$$\tau_{xz} = \tau_{yz} = O(r^o)$$

In equations (32), r and θ are the local polar coordinates with origin at the crack point x=a and y=o in the xy-plane. symbol $\Phi(1)$ is the value of the function $\Phi(\xi)$ as $\Phi\to 1$, the location of the normalized crack tip. The functions $\sigma_p(z)$ and $\sigma_T(z)$ are defined by the expressions

$$\sigma_p(z) = \frac{M}{\kappa} f_1''(z)p_o$$

$$\sigma_T(z) = -\frac{N}{\kappa} f_1(z)p_o \qquad (33)$$

in which

$$M = (1-\frac{\gamma_2}{2}\delta_3)^{-1} [(I_1+I_2) \frac{\delta_3}{2} (\delta_5\delta_2+\delta_6\delta_1) + \frac{I_3}{I_1-I_2} \frac{\delta_2\delta_6}{2a_6}]$$

$$N = (1-\frac{\gamma_2}{2}\delta_3)^{-1} [\delta_3(\delta_2\delta_1+\delta_1\delta_2)] \tag{34}$$

The customary inverse square root singularity at the crack front and the angular variations of the stresses are observed in equations (32). This is in agreement with the exact three-dimensional crack edge solution [4]. The approximation of this theory enters only through the intensity of the local stress field. In this connection, the stress intensity factor may be defined by

$$k_1(z) = K\Phi(1)\ \sigma_p(z)a^{1/2} \tag{35}$$

for the in-plane stresses and

$$k_1(z) = K\Phi(1)\ \sigma_T(z)a^{1/2} \tag{36}$$

for the transverse stress σ_z. Up to this point, the in-plane behaviors of the stresses are completely determined. Additional conditions based on the characteristics of the stresses in the interior region of the layer and near the free surface on interface are required for the determination of the functions $f_1(z)$. Photoelastic Analysis [4] has shown that the stresses in the interior of plate are nearly in a state of plane strain while they undergo large variations in a layer near the plate surface. Substituting equations (32) and (33) into the plane strain condition yields

$$f_1(z) = A \cos (p_1z) \tag{37}$$

The parameter p_1 controls the qualitative feature of the stress distribution across the layer thickness and has the form

$$p_1 = \left(\frac{N}{2\nu_1 M}\right)^{1/2} \tag{38}$$

The constant A in equation (37) serves as a quantitative measure of the stresses.

Nonsingular Stress Distribution Across The Laminate. It is observed from equations (5) that the stresses in the laminate can be found by multiplying the appropriate functions $f_p(z)$ or its derivatives with the in-plane quantities S_x, S_y, etc., defined in equations (30). Since S_x, S_y, etc. may be evaluated by numerical integration and the numerical solution of the Fredholm integral equation (38), the task here is to construct the functions $f_2(z)$ and $f_3(z)$ that are consistent with equations (3) and (37). To this end, the following expressions for $f_p(z)$ may be written:

$$f_1(z) = A \cos(p_1 z), \quad 0 \leq z \leq h_1$$

$$f_2(z) = A\{\cos(p_1 h_1)\cos[p_2(z-h_1)] - \frac{p_1}{p_2}\sin(p_1 h_1)$$
$$\sin[p_2(z-h_1)]\}, \quad h_1 \leq z \leq h_1+h_2 \tag{39}$$

$$f_3(z) = A\{[\cos(p_1 h_1)\cos(p_2 h_2) - \frac{p_1}{p_2}\sin(p_1 h_1)\sin$$
$$(p_2 h_2)]\cos[p_3(z-h_1-h_2)] - \frac{1}{p_3}[p_2\cos(p_1 h_1)\sin$$
$$(p_2 h_2) + p_1\sin(p_1 h_1)\cos(p_2 h_2)]\sin[p_3$$
$$(z-h_1-h_2)]\}, \quad h_1+h_2 \leq z\ h_1+h_2+h_3$$

The parameters p_j, $j = 1,2,3$, are defined as

$$p_j = \left(\frac{N}{2\nu_j M}\right)^{1/2} \tag{40}$$

With these definitions for $f_p(z)$, the stress field in an arbitrary point in the laminate may be obtained from the following expressions:

$$\sigma_x(x,y,a) = \frac{2Kp_o a^2}{\kappa} f_p''(z) \int_0^\infty \frac{\psi(s)}{g_1(s)} \mathrm{Re}\{I_1 s^2 + I_2 \rho^2) \frac{(1+i\beta)}{I_1^2 - I_2^2}$$

$$[(\delta_1 - i\delta_2)I_3 - 1]e^{-\rho y} - (\delta_1 - i\delta_2)(1+i\beta)(\gamma_1 - i\gamma_2)$$

$$\frac{I_4}{I_1+I_2} e^{-\rho y} - (\delta_1 - i\delta_2)(1+i\beta) \frac{q\rho I_3}{I_1 - I_2} e^{-qy}\}\cos(sx)ds$$

$$\sigma_y(x,y,z) = \frac{2Kp_o a^2}{\kappa} f_p''(z) \int_0^\infty \frac{\psi(s)}{g_1(s)} \mathrm{Re}\{(I_2 s^2 + I_1 \rho^2) \frac{(1+i\beta)}{I_1^2 - I_2^2}$$

$$[1 - (\delta_1 - i\delta_2)I_3]e^{-\rho y} - (\delta_1 - i\delta_2)(1+i\beta)(\gamma_1 - i\gamma_2)$$

$$\frac{I_4}{I_1+I_2} e^{-\rho y} + (\delta_1 - i\delta_2)(1+i\beta) \frac{q\rho I_3}{I_1 - I_2} e^{-qy}\}\cos(sx)ds$$

$$\tau_{xy}(x,y,z) = \frac{2Kp_o a^2}{\kappa} f_p''(z) \int_0^\infty \frac{\psi(s)}{g_1(s)} \mathrm{Re}\{\frac{\rho s}{I_1 - I_2} [(1+i\beta)$$

$$\langle 1 - (\delta_1 - i\delta_2)I_3 \rangle . e^{-\rho y} + (\delta_1 - i\delta_2)(1+i\beta) \frac{I_3(s^2+q^2)}{2s^2}$$

$$e^{-qy}]\}\sin(sx)ds$$

$$\tau_{xz}(x,y,z) = \frac{-2Kp_o a^2}{\kappa} f_p'(z) \int_o^\infty \frac{\psi(s)}{g_1(s)} \mathrm{Re}\{(\delta_1 - i\delta_2)(1+i\beta)$$

$$(se^{-\rho y} - \frac{q\rho}{s} e^{-qy})\}\sin(sx)ds \tag{41}$$

$$\tau_{yz}(x,y,z) = \frac{-2Kp_o a^2}{\kappa} f_p'(z) \int_o^\infty \frac{\psi(s)}{g_1(s)} \mathrm{Re}\{(\delta_1 - i\delta_2)(1+i\beta)$$

$$(e^{-\rho y} - e^{-qy})\}\cos(xs)ds$$

$$\sigma_z(x,y,z) = \frac{2Kp_o a^2}{\kappa} f_p(z) \int_o^\infty \frac{\psi(s)}{g_1(s)} \mathrm{Re}\{(\delta_1 - i\delta_2)(1+i\beta)$$

$$(\gamma_1 - i\gamma_2)e^{-\rho y}\}\cos(sx)ds$$

Because of assumptions (2,), (3) and (4) the stress expressions (41) are not expected to give satisfactory results in the regions bounded by $x=a$, $y=o$ throughout the laminate except on the middle layer that contains the crack. Moreover, as mentioned in [1] and [5] the plane strain condition, which leads to the construction of the functions $f_p(z)$, are not expected to hold near a free surface or interface because of the lack of mechanical constraint. Hence, $f_p(z)$ must be constructed separately whenever the condition $\sigma_z = \nu(\sigma_x + \sigma_y)$ is not satisfied. The concept of a boundary layer can then be introduced. Such a procedure results in additional discussions and is beyond the scope of the present investigation.

NUMERICAL RESULTS AND DISCUSSIONS

The numerical results in this report are divided into two parts. The first pertains to the five-layer laminated composite where attention is focused on the variations of the stress intensity factor with the material geometrical parameters. The second considers the stress distribution in a laminated plate due to the influence of adhesive layers. This is done for a specific example of aluminum-epoxy-steel system. The stress intensity factor as well as stress field away from the crack are determined for this example. For convenience, the Poisson's ratios for all the numerical computations are taken to be $\nu_1 = \nu_2 = \nu_3 = 0.3$.

The Five-Layer Problem. The stacking sequence of a laminate is determined by assigning specific values of the material and geometrical parameters and will affect the intensity of the stresses transmitted to the crack front region. The stress intensity factor gave a measure of this influence and in its normalized form is given by

$$\bar{k}_1 = \frac{k_1(z)}{\sigma_p(z)a^{1/2}} = K\Phi(1) \tag{42}$$

in which K is given in equation (13) and $\Phi(1)$ may be calculated from the Fredholm integral equation (28). The variation of $\bar{k}_1$ as a function of the center layer thickness to crack length ratio h_1/a, is plotted in Fiqure 2 for various values of $P = p_1 h_1$. The Young's modulus and thickness ratio are taken to be

$E_2/E_1 = 0.1$, $E_3/E_1 = 1.0$, $h_2/h_1 = 0.1$ and $h_3/h_1 = 1.0$. The choice of P depends on the transverse stress distribution in a given problem. Numerically, it is confined to $0<P<\frac{\pi}{2}$ because of the cos (P) dependence of the stress field which changes sign when P is outside of the above range. Note that as P is decreased, $\bar{k}_1$ rises more sharply for small values h_1/a. This type of behavior was also observed for the three-layer problem [1].

Figure 3 shows a plot of $\bar{k}_1$ versus h_1/a for $E_3/E_1 = 0.1$, 1.0 and 10.0. The parameter P is chosen to be 0.7 while $E_2/E_1 = 0.1$, $h_2/h_1 = 0.1$ and $h_3/h_1 = 5.0$. The stress intensity factor increases with decreasing stiffness ratio E_3/E_1. The same trend is observed in Figure 4 and 5 where the thickness ratio h_3/h_1 is taken to be 1.0 and 0.2, respectively. By comparing the above three figures, it is evident that the stress intensity factor increases with increasing h_3/h_1 for $E_3/E_1<1.0$. On the other hand, the opposite trend is observed for $E_3/E_1>1.0$ This behavior is more clearly illustrated in Figure 6 where the variations of $\bar{k}_1$ versus h_3/h_1 for three E_3/E_1 ratio are plotted. The values of the other parameters are fixed at $P = 0.7$, $E_2/E_1 = 0.1$, $h_2/h_1 = 0.1$ and $h_1/a = 1.0$. Thus the crack tip stress can be lowered by (a) raising the stiffness ratio E_3/E_1, or (b) decreasing the thickness ratio h_3/h_1 for $E_3/E_1<1.0$, or (c) increasing the h_3/h_1 ratio for $E_3/E_1>1.0$. These conclusions are valid as long as the parameters in layer number two are kept constant.

The variations of $\bar{k}_1$ with h_1/a are displayed in Figure 7 for fixed values $P = 0.7$, $E_3/E_1 = 1.0$, $h_3/h_1 = 1.0$ and $h_2/h_1 = 5.0$. The three curves represent stiffness ratios of $E_2/E_1 = 0.1$, 1.0 and 10.0. Hence, $\bar{k}_1$ decreases with increasing E_2/E_1 ratio. Similar trends are illustrated in Figure 8 for $h_2/h_1 = 1.0$ and in Figure 9 for $h_2/h_1 = 0.1$. The effect of the thickness ratio h_2/h_1 on the stress intensity factor is demonstrated in Figure 10 for $P = 0.7$, $E_3/E_1 = 1.0$, $h_3/h_1 = 1.0$, $h_1/a = 1.0$. Therefore, the same conclusion as that for the outside layer can be drawn here. The stress intensity factor decreases with (a) increasing E_2/E_1 ratio, or (b) decreasing h_2/h_1 ratio if $E_2/E_1<1.0$, or (c) increasing h_2/h_1 ratio if $E_2/E_1>1.0$.

Cracked Laminate With Adhesive Layers. As mentioned before, the five-layer plate problem may be used to model the presence of adhesive layers in laminated structures. Let material 2 in Figure 1 represent the adhesive layers. As a specific example, numerical computations were performed on an aluminum-epoxy-steel system. Layer one is assumed to be made of aluminum, layer two of epoxy and layer 3 of steel. Therefore, the stiffness ratios are $E_3/E_1 \approx 3.0$ and $E_2/E_1 = 0.05$. The thickness ratio h_3/h_1 is taken to be one. Since the adhesive layer is about one order of magnitude smaller than the other layers, the thickness ratio h_2/h_1 should be such that $0<h_2/h_1\leq 0.1$. Within this range, the scales of Figure 10 show

that h_2/h_1 has little effect on the stress intensity factor. This is shown by the $\bar{k}_1$ versus h_2/h_1 in Figure 11 for the aluminum-epoxy-steel system. The crack is in the aluminum layer. The parameters are fixed at $P = 0.7$, $h_1/a = 1.0$ and $\nu_1 = \nu_2 = \nu_3 = 0.3$.

It should be said that although the adhesive layers have negligible influence in the stress intensity factor, their presence may affect inter layer separation or delmaination. For this reason, it is essential to turn to the calculation of stress distribution in the adhesive layers.

Define the normalized stress $\bar{\sigma} = \sigma/\sigma_o$ where σ_o is the applied stress. Figure 12 depicts the variations of the stresses as with the normalized thickness parameter z/h_1 for $h_2/h_1 = 0.0$ or for a three-layer laminate. The stresses are computed at the point $x/a = 1.0$ and $y/a = 3.0$. It is seen that $\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\sigma}_z$ and $\bar{\tau}_{xy}$ decrease in magnitude as z/h_1 is increased. The maximum value of these stresses occur at the center of the plate, i.e., $z = 0$. For the transverse shear stresses $\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$, the trend is reversed. A similar plot is Figure 13 for $h_2/h_1 = 0.1$ exhibits the same behavior except the magnitudes are slightly smaller than their counterpart in Figure 12. In Figure 14, variations of the stresses for $h_2/h_1 = 1.0$ are displayed. Unlike the previous two cases for $h_2/h_1 = 0.0$ and 0.1, $\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\sigma}_{xy}$ and $\bar{\sigma}_z$ changes sign when the free surface is approached. Figure 15 depicts the stress variation

across the thickness at a different point $x/a = 3.0$ and $y/a = 3.0$. This point is further away from the crack as compared with the point in the previous figures. For $h_2/h_1 = 0.0$, the stress components $\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\tau}_{xy}$ and $\bar{\sigma}_z$ decrease in magnitude as z/h_1 increases. However, $\bar{\sigma}_y$ becomes compressive. $\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$ again rises with increasing z/h_1 except $\bar{\tau}_{xz}$ has negative sense while $\bar{\tau}_{yz}$ is positive. Similar plot for $h_2/h_1 = 0.1$ is shown in Figure 16 for comparison. Figure 17 displays the variation of $\bar{\sigma}_j$ versus z/h_1 for $h_2/h_1 = 1.0$. It is interesting to note that the transverse shear stresses reach a peak and then drop off. Figure 12-17 give some insight into the stress distribution across the thickness in the laminate.

To illustrate the stress variation in the xy-plane at a given z elevation, three-dimensional plots of the normal stresses $\bar{\sigma}_x$, $\bar{\sigma}_y$ and $\bar{\sigma}_z$ are depicted, respectively, in Figures 18, 19 and 20 as functions of the position parameters x/a and y/a for $z/h_1 = 1$. These stresses are thus corresponding to the interfacial stresses between layer one and three since $h_2/h_1 = 0$ in these figures. It is observed that all the stresses die out quickly with increasing x/a and y/a values. All three stress components changes sign from positive to negative before reaching zero asymptotically.

More detailed stress variations as a function of x/a, y/a, z/h, and h_2/h_1 are tabulated in Tables 1-9. Table 1 shows the stress field for a three-layer laminate at the middle of the

center layer. The parameters x/a and y/a are extended from 1.0-5.0 and 1.0-10.0, respectively. No need to expand these ranges since the stresses have already reached their asymptotic values. Table 2 exhibits the stresses at the interface. The stress field in the middle of the outer layer is listed in Table 3. By taking $h_2/h_1 = 0.02$, Table 4-6 shows the stress distribution as a function of x/a and y/a at the middle of the center layer, of the adhesive layer and of the outside layer. Finally, Table 7-9 lists the variations of the stresses for $h_2/h_1 = 0.1$ at the middle of each layer.

To recapitulate, the stress calculations present here are essential to the prediction of failures by the delamination mechanism. A failure criterion must be used in conjunction with these stress results to forecast prediction. This phase of the work will be left for future research.

CONCLUSIONS

A method for computing the stress distributions in a five-layered composite plate was developed. By knowing the fracture toughness, k_{1c}, of the middle layer which contains the crack the load at instability may be estimated. To check for delamination, the stresses obtained in the adhesive layers may be used to compute for the strain energy density factor, S [6]. The location of minimum S or S_{min} predicts where failure may occur while the critical value of S_{min}, S_{cr}, indicates when failure may occur. The critical load obtained in this way may then be compared with that found from the critical stress intensity factor calculation to determine whether or not delamination has occurred before crack propagation.

On basis of the aluminum-epoxy-steel laminate example, several points of interest may be concluded:

(1) In the case of a five-layered laminate, the stiffness ratios E_2/E_1 and E_3/E_1, and/or the thickness ratios h_2/h_1 and h_3/h_1 may be combined in such a way to lower the crack front stress intensity. This can elevate the allowable design stress and/or prolong the life expectancy of the composite structure.

(2) The adhesive layers in the laminate do not appear to have a significant influence on the stress intensity factor for adhesives in the practical thickness ratio range of $0<h_2/h_1\leq 0.1$.

(3) The present scheme of stress analysis coupled with a realistic fracture criterion may be stored in the computer for otimizing the design of laminates against brittle fracture.

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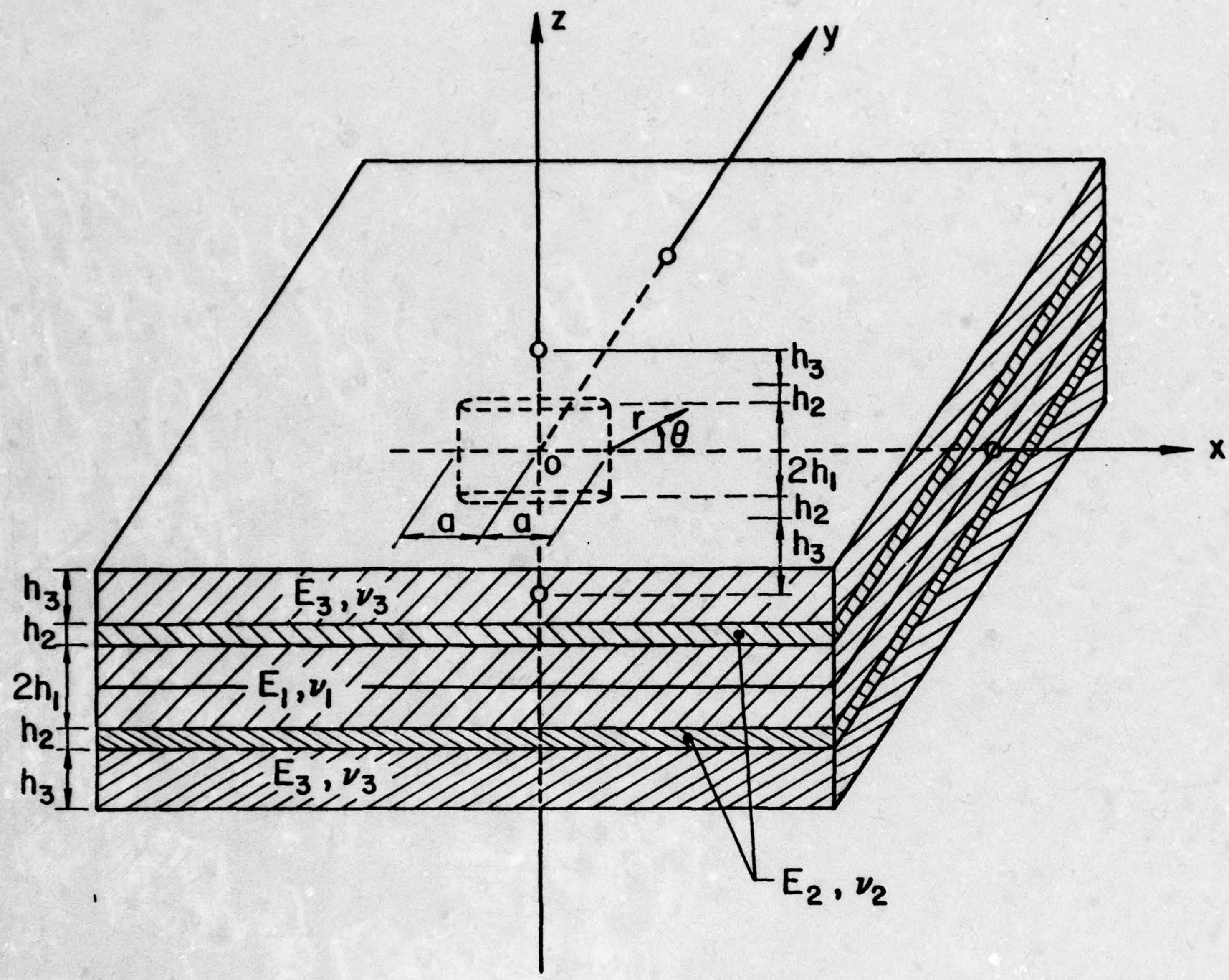

Figure 1. - Laminated Composite Plate With An Embedded Crack

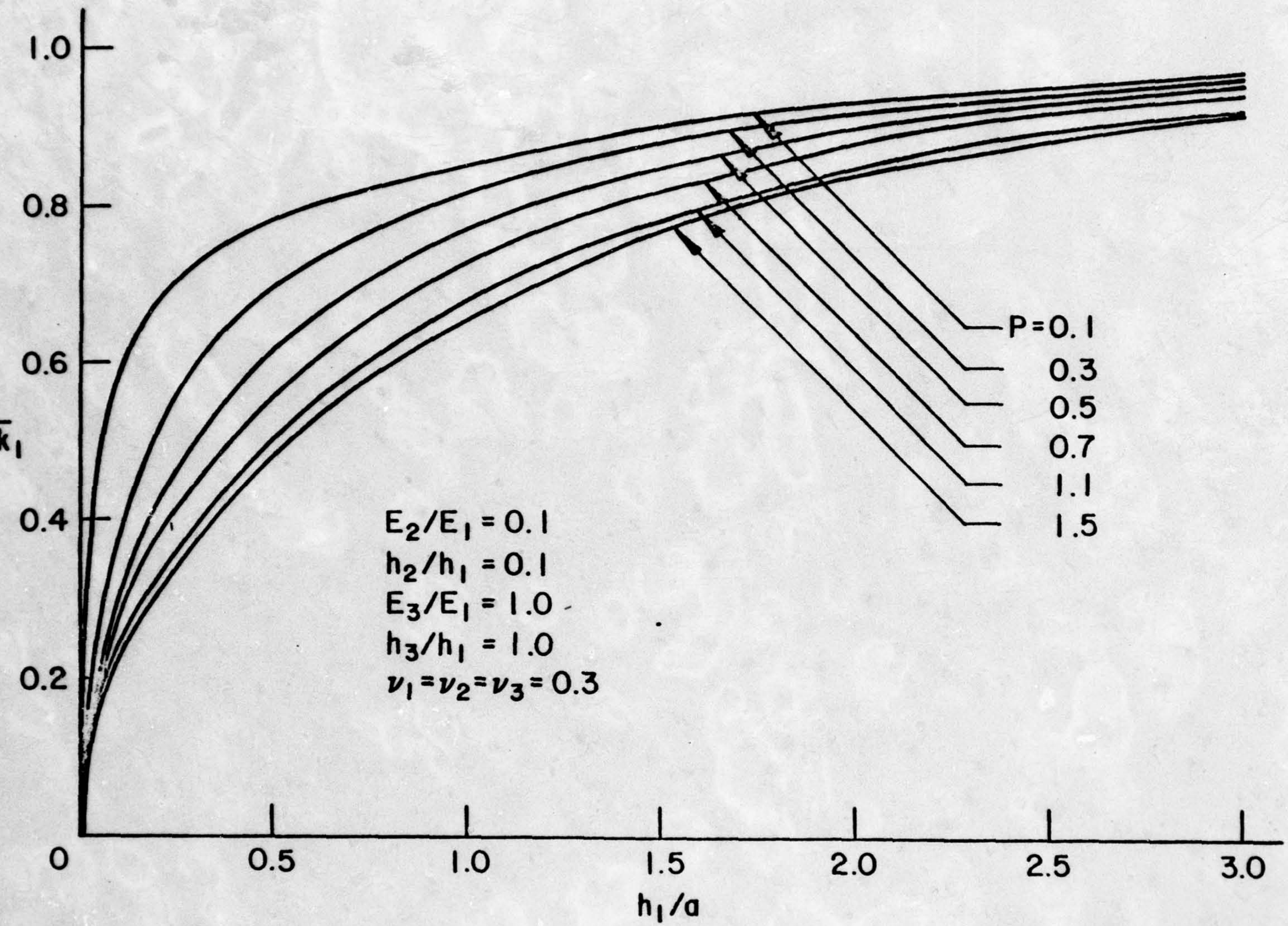

Figure 2. - $\bar{k}_1$ vs. h_1/a for various values of P

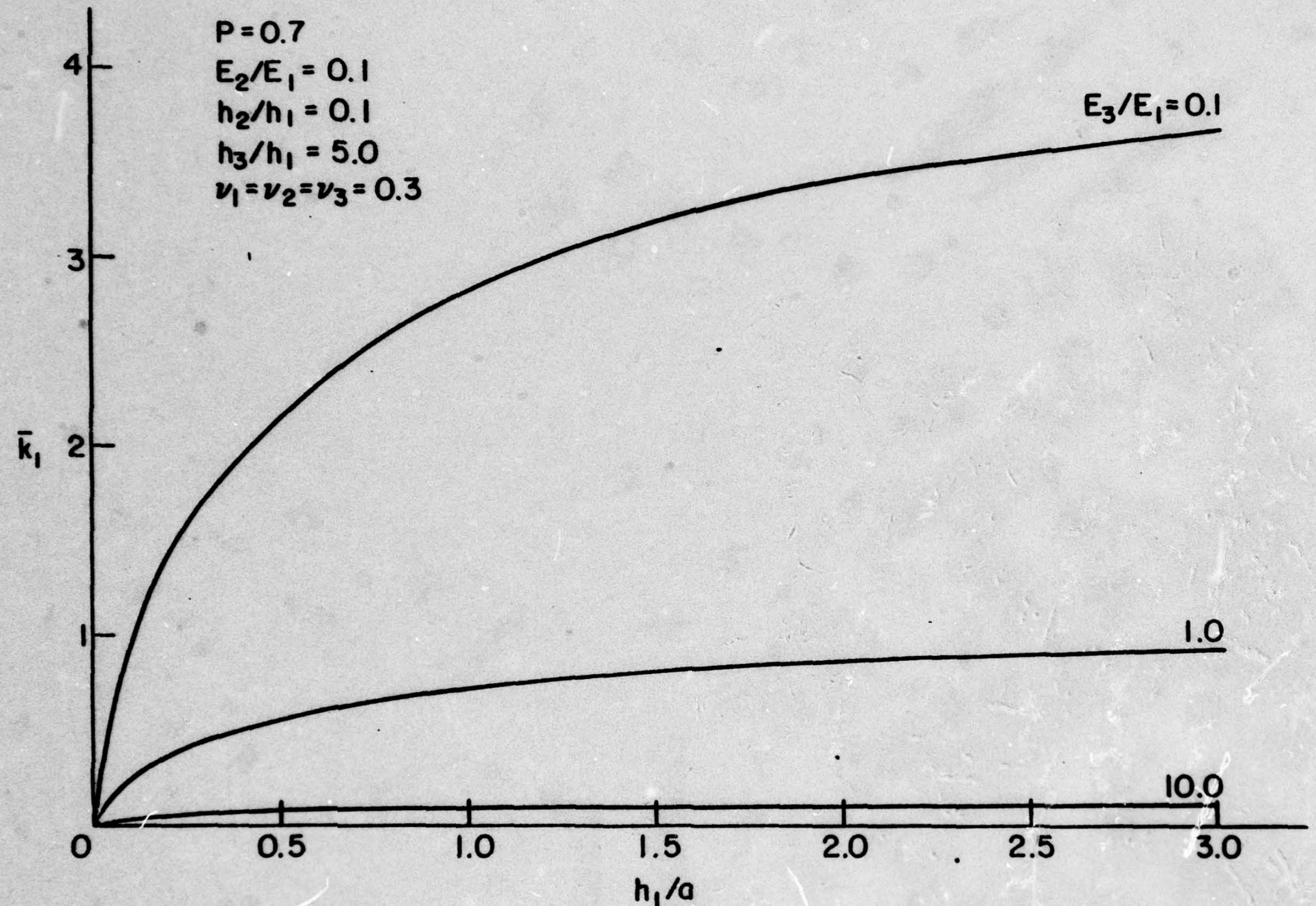

Figure 3. - $\bar{k}_1$ vs. h_1/a for various ratios of E_3/E_1 (h_3/h_1 = 5.0)

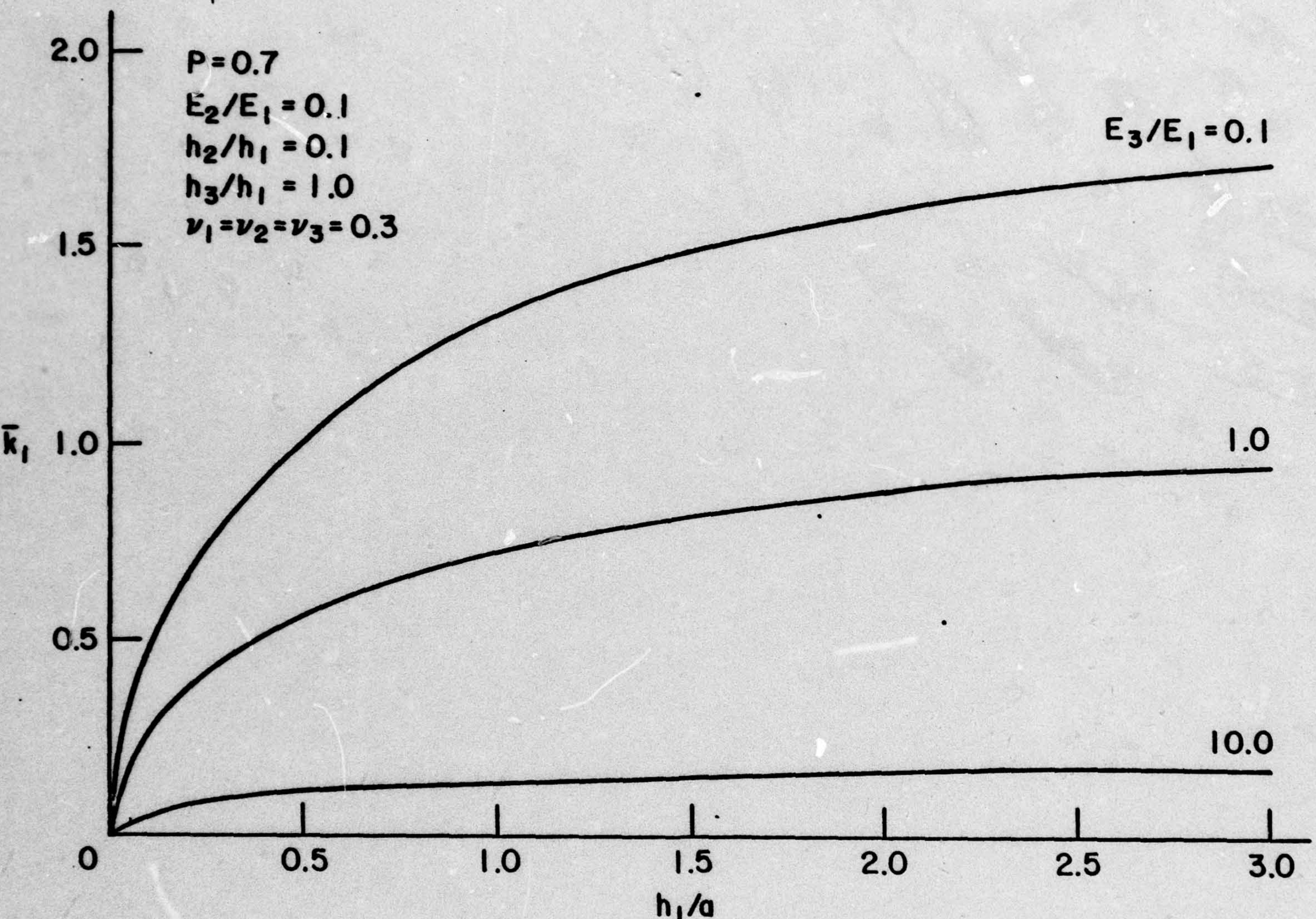

Figure 4. - $\bar{k}_1$ vs. h_1/a for various ratios of E_3/E_1 (h_3/h_1 = 1.0)

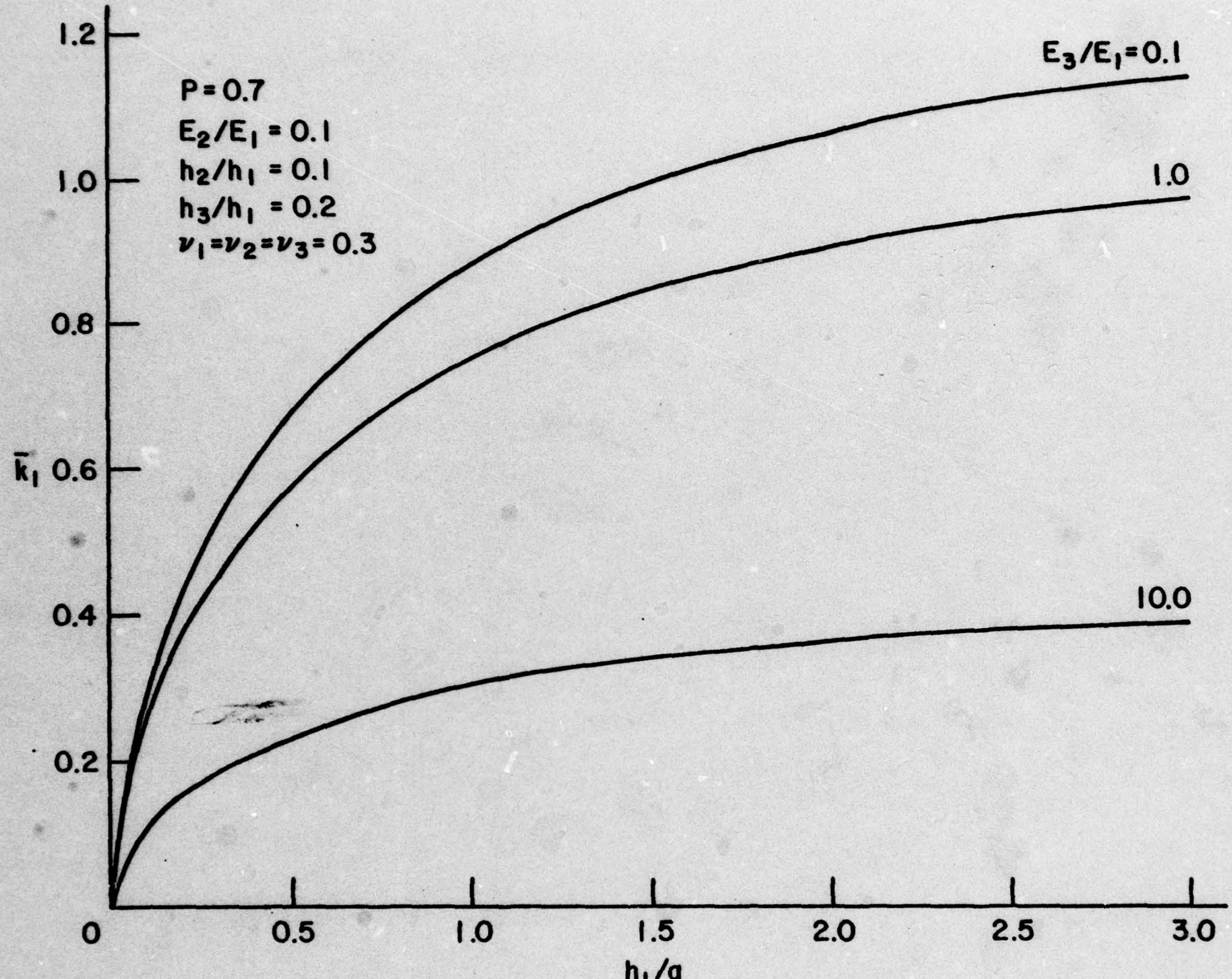

Figure 5. - $\bar{k}_1$ vs. h_1/a for various ratios of E_3/E_1 ($h_3/h_1 = 0.2$)

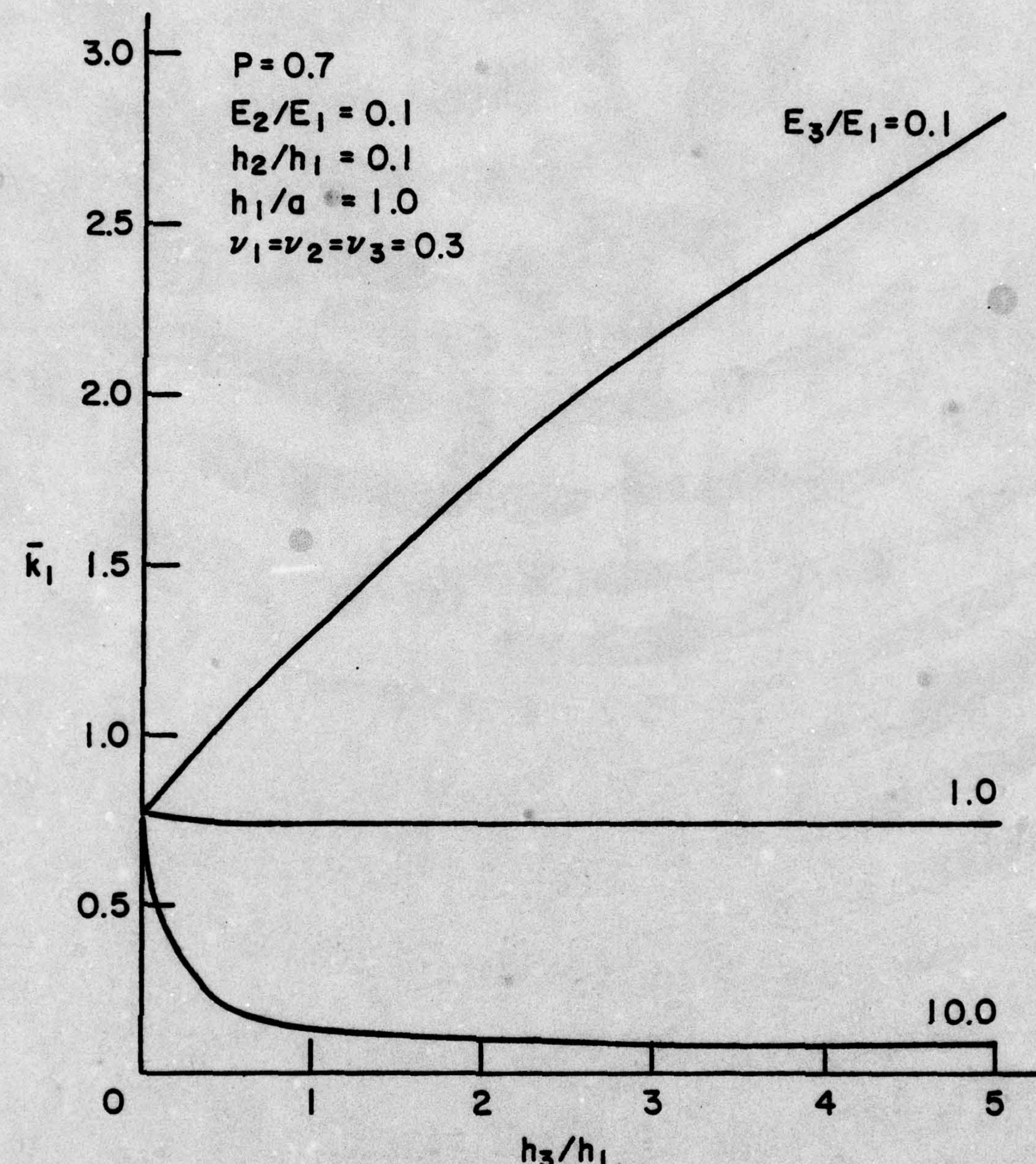

Figure 6. - $\bar{k}_1$ vs. h_3/h_1 for various E_3/E_1 ratios

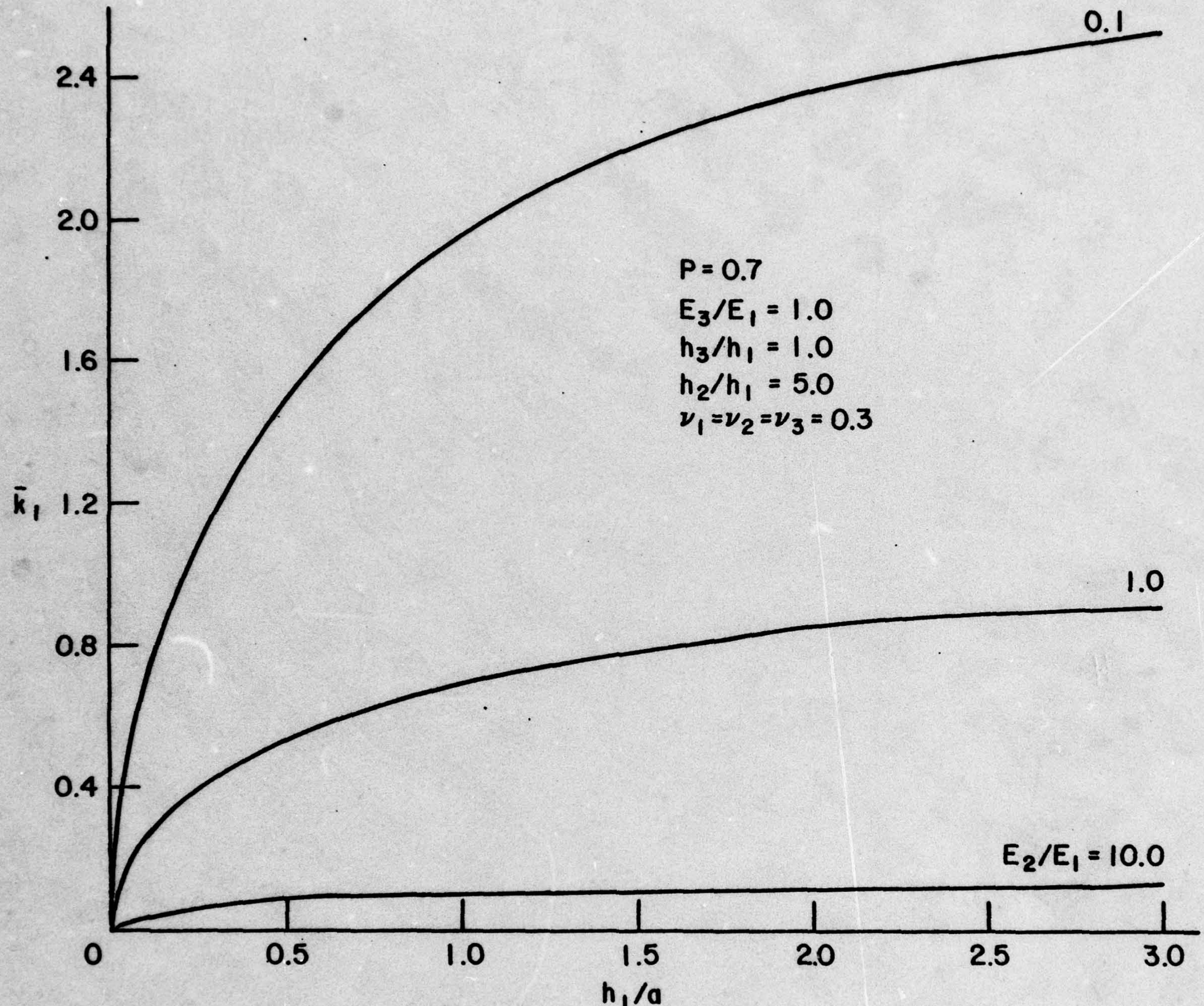

Figure 7. - $\bar{k}$ vs. h_1/a for various ratios of E_2/E_1 $(h_2/h_1 = 5.0)$

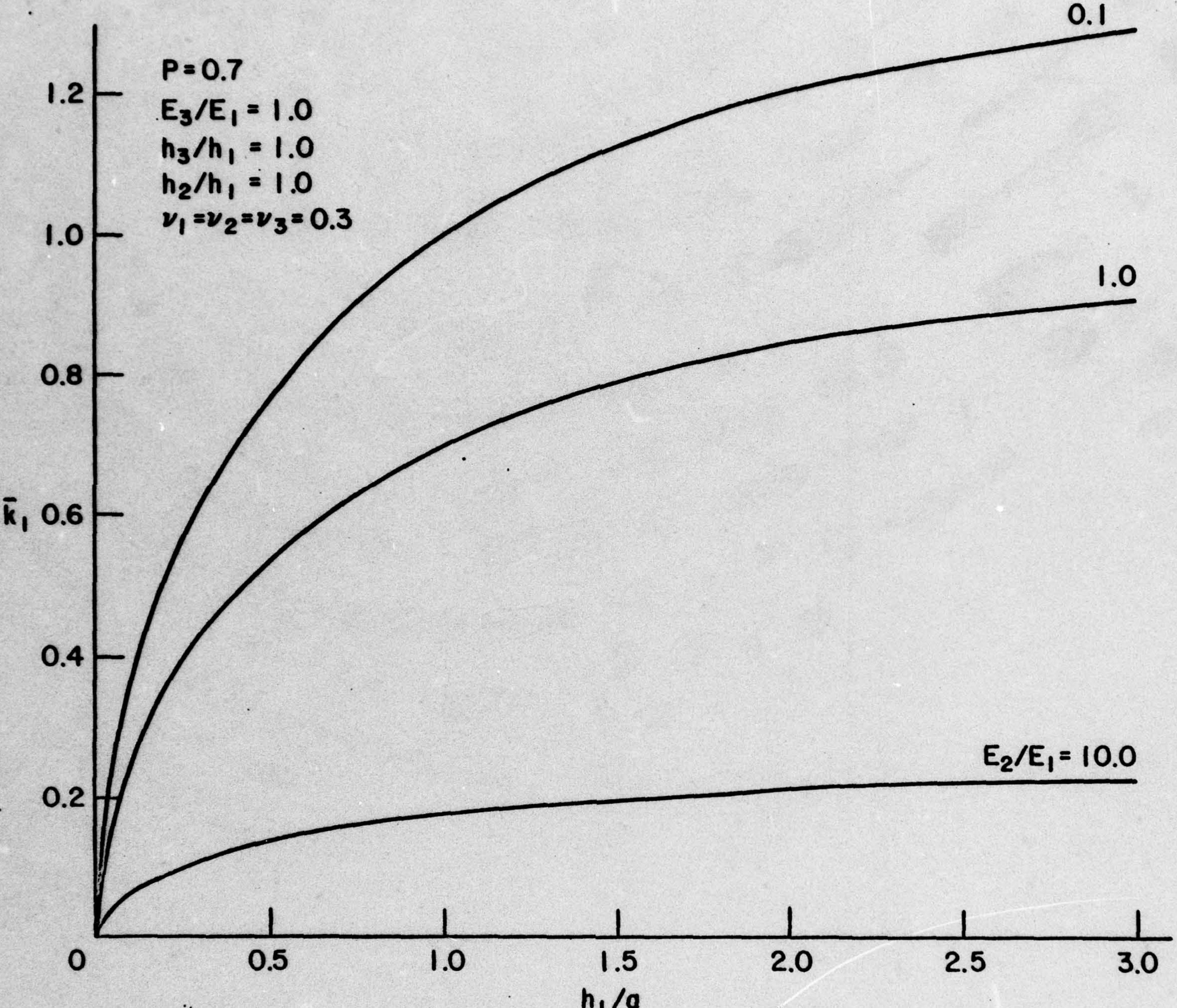

Figure 8. - $\bar{k}_1$ vs. h_1/a for various ratios of E_2/E_1 (h_2/h_1 = 1.0)

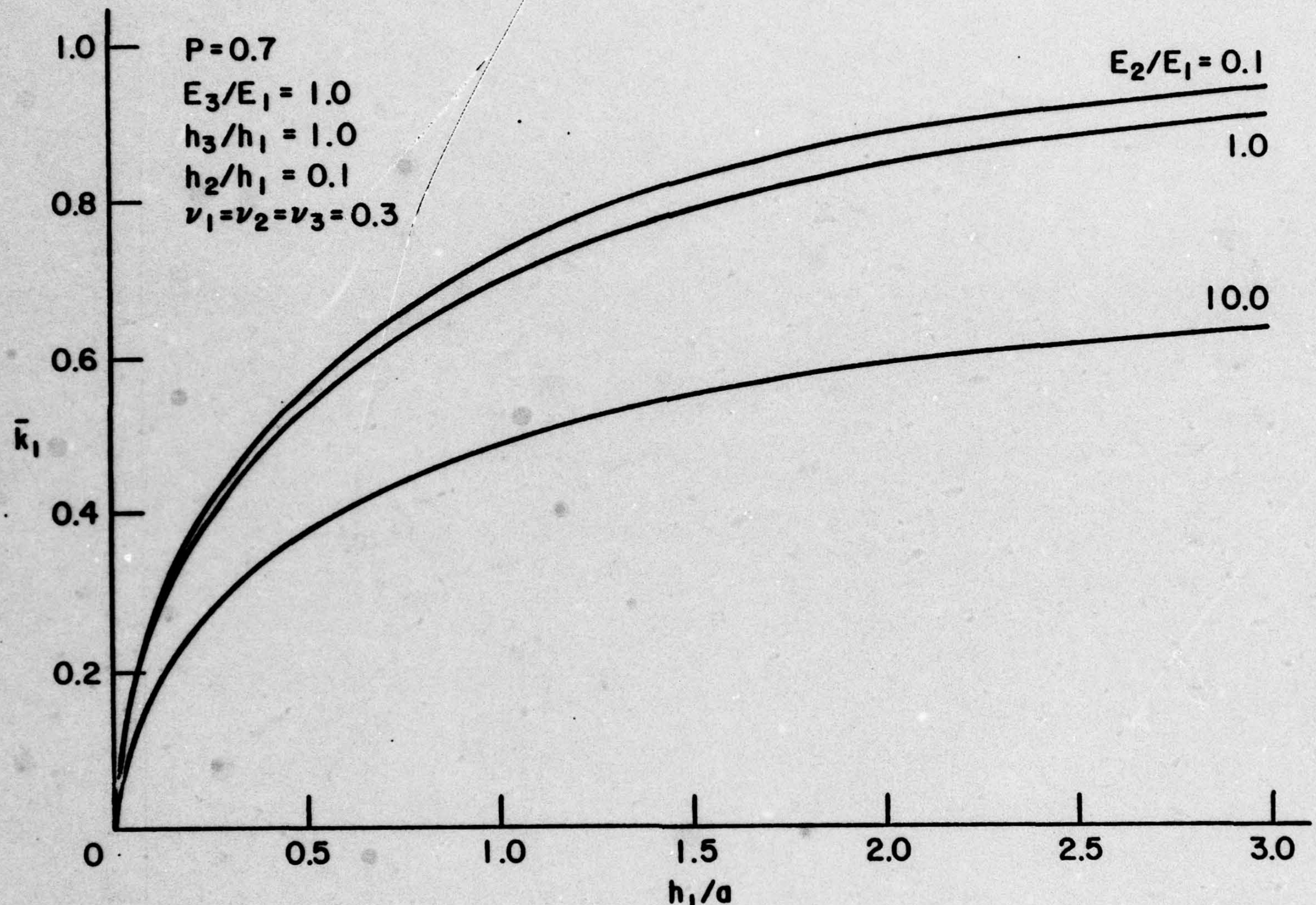

Figure 9. - $\bar{k}_1$ vs. h_1/a for various ratios of E_2/E_1 ($h_2/h_1 = 0.1$)

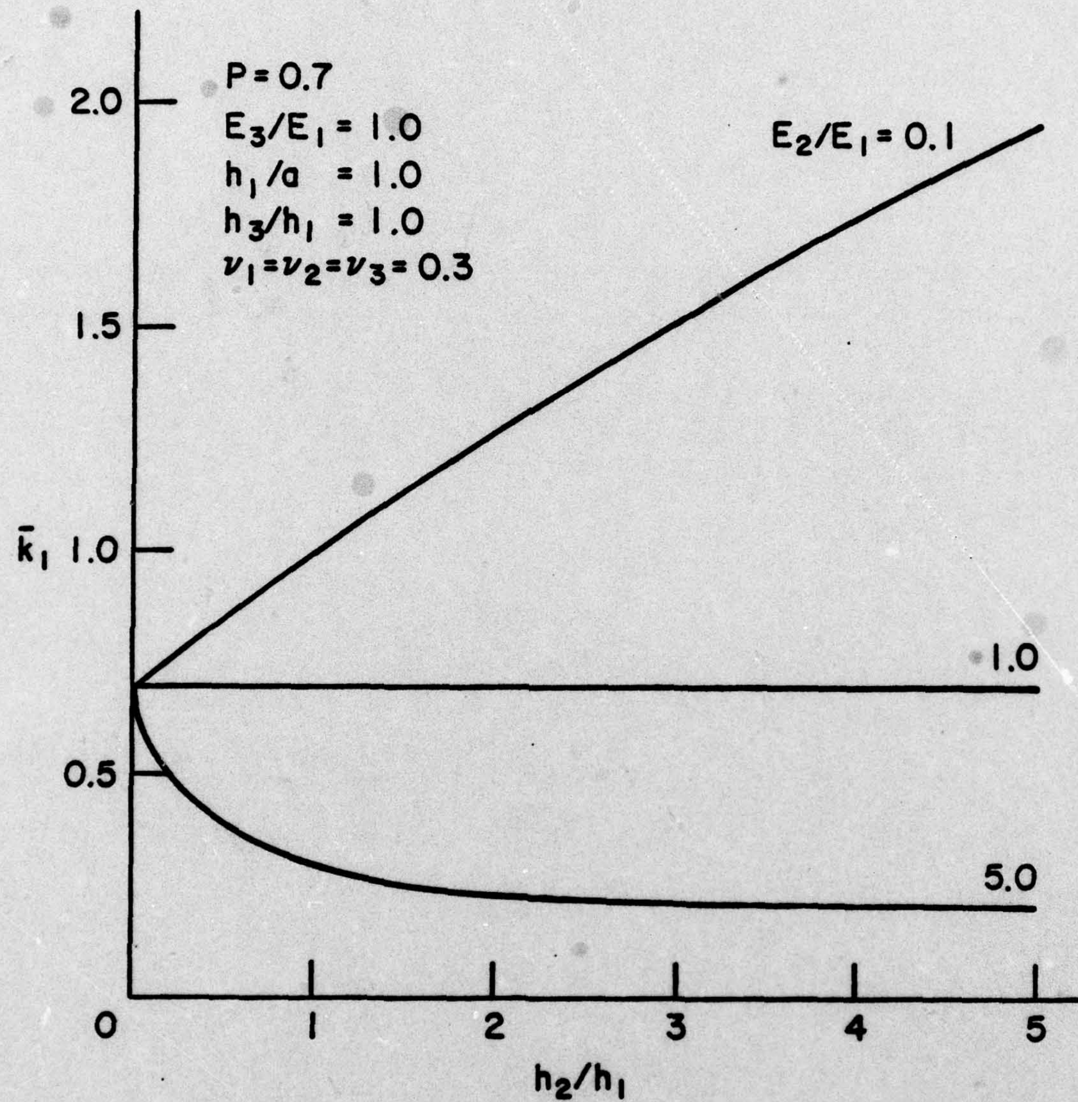

Figure 10. - $\bar{k}_1$ vs. h_2/h_1 for various E_2/E_1 ratios

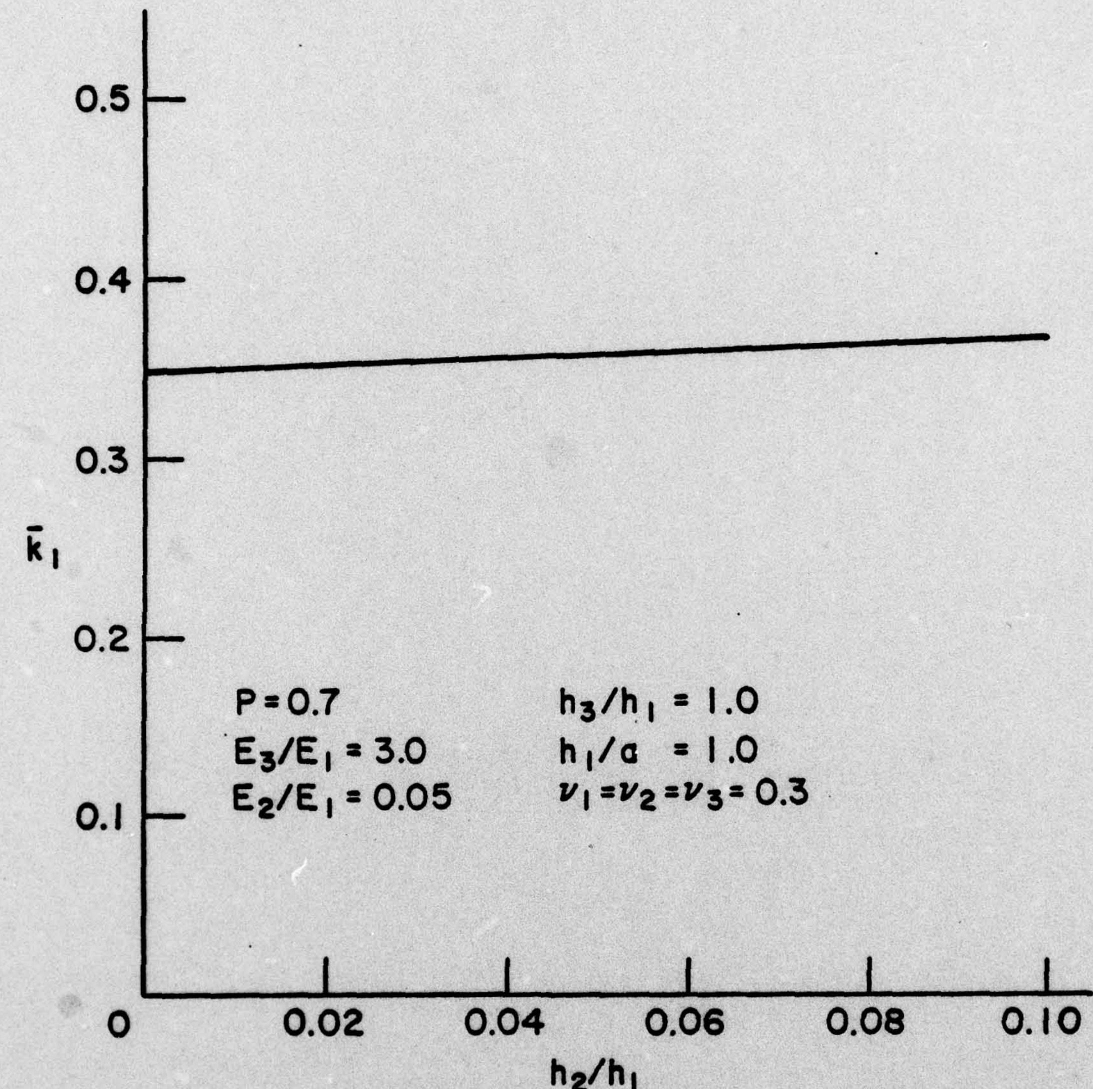

Figure 11. - $\bar{k}_1$ vs. h_2/h_1 for small h_2/h_1

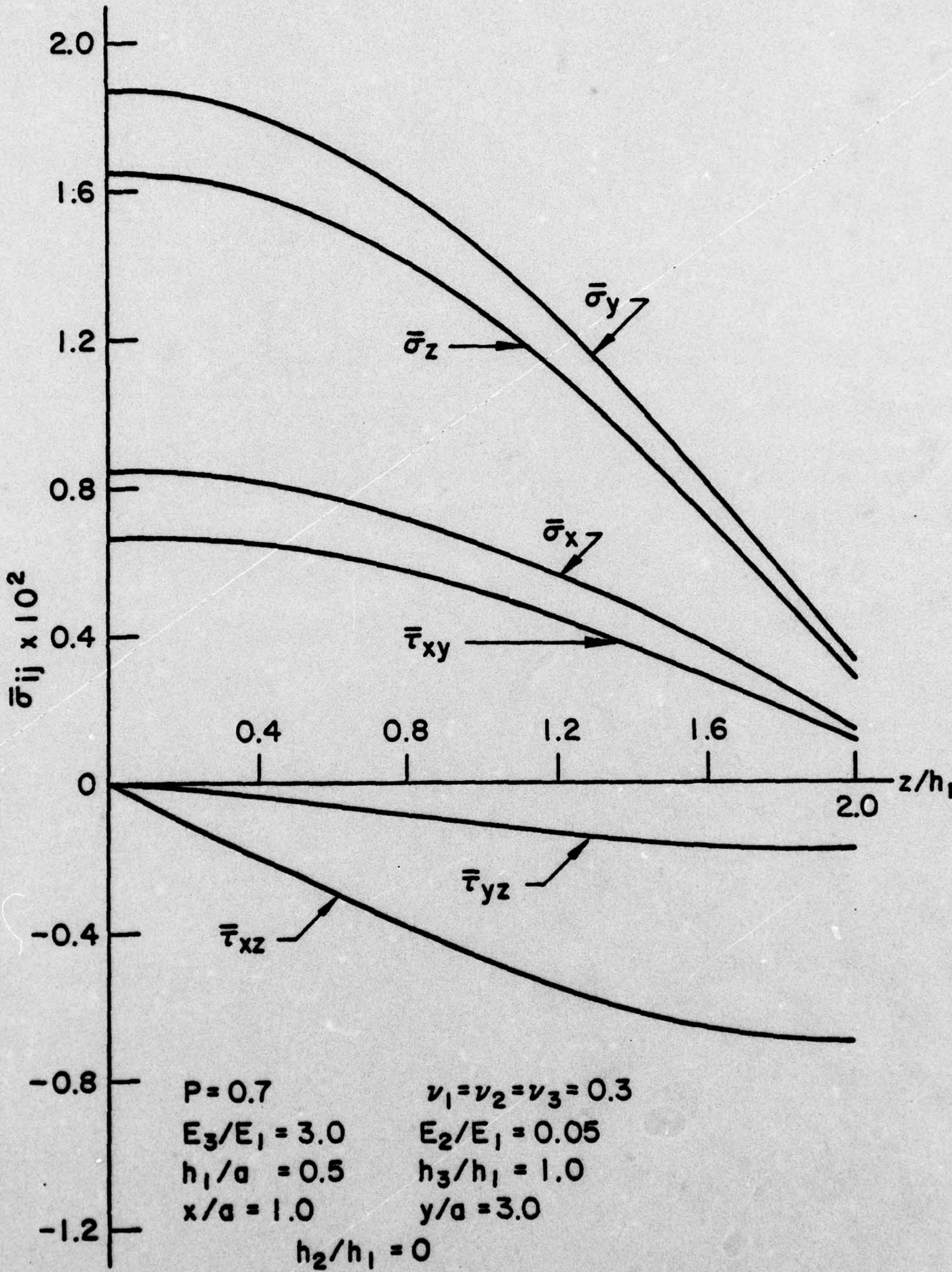

Figure 12. - Variation of stresses across the thickness for $h_2/h_1 = 0.0$ ($x/a = 1.0$, $y/a = 3.0$)

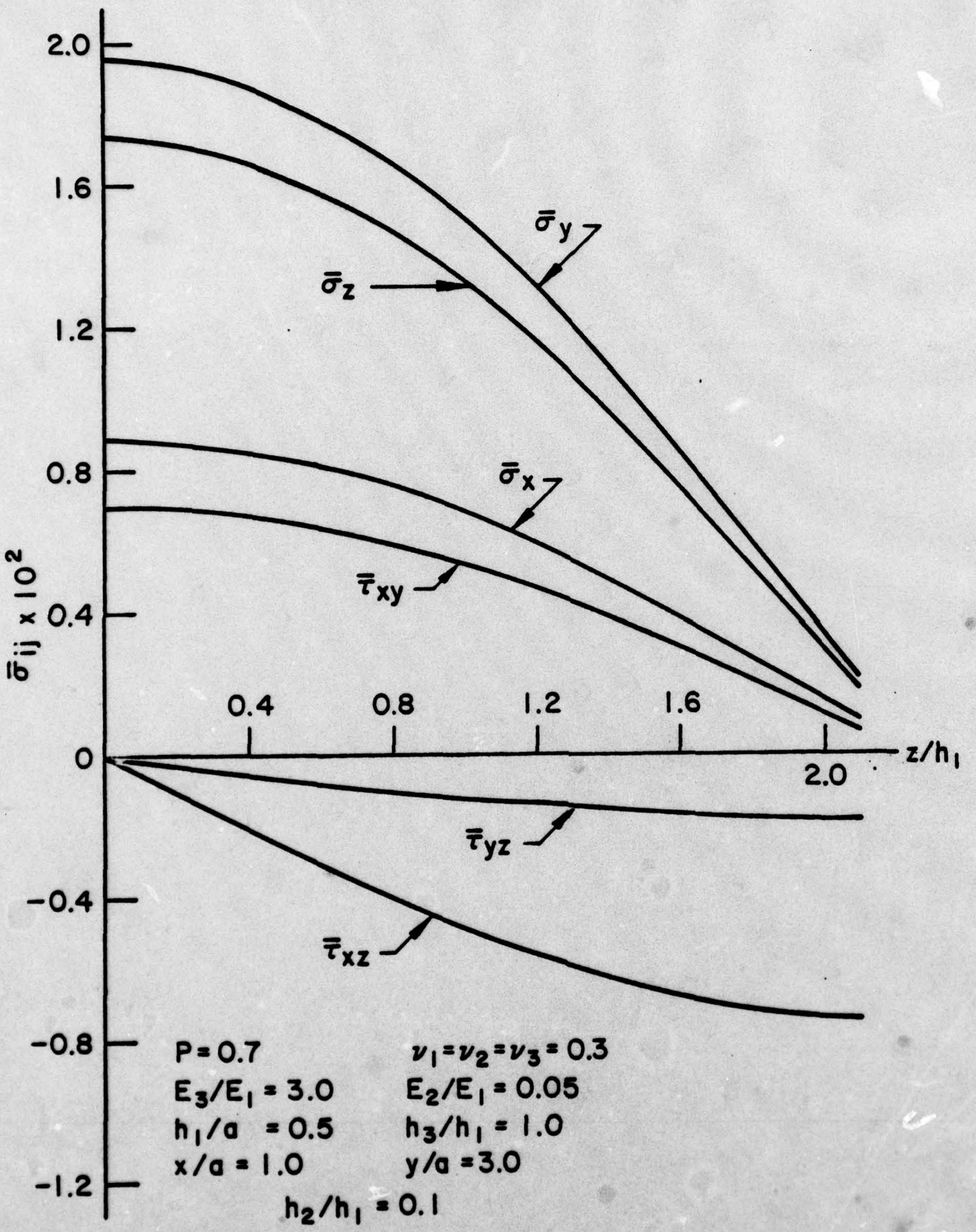

Figure 13. - Variation of stresses across the thickness for $h_2/h_1 = 0.1$ ($x/a = 1.0$, $y/a = 3.0$)

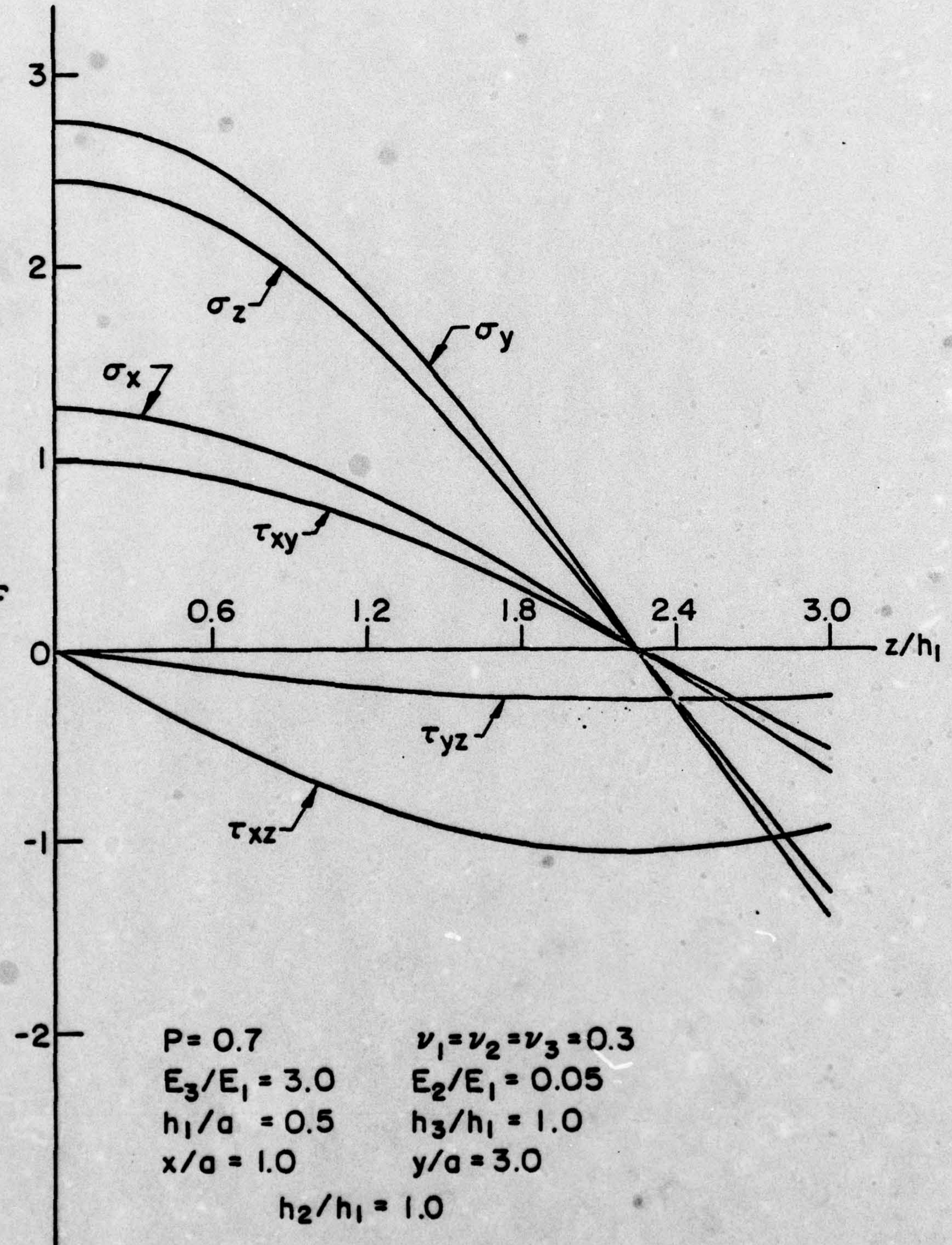

Figure 14. - Variation of stresses across the thickness for h_2/h_1 = 1.0 (x/a = 1.0, y/a = 3.0)

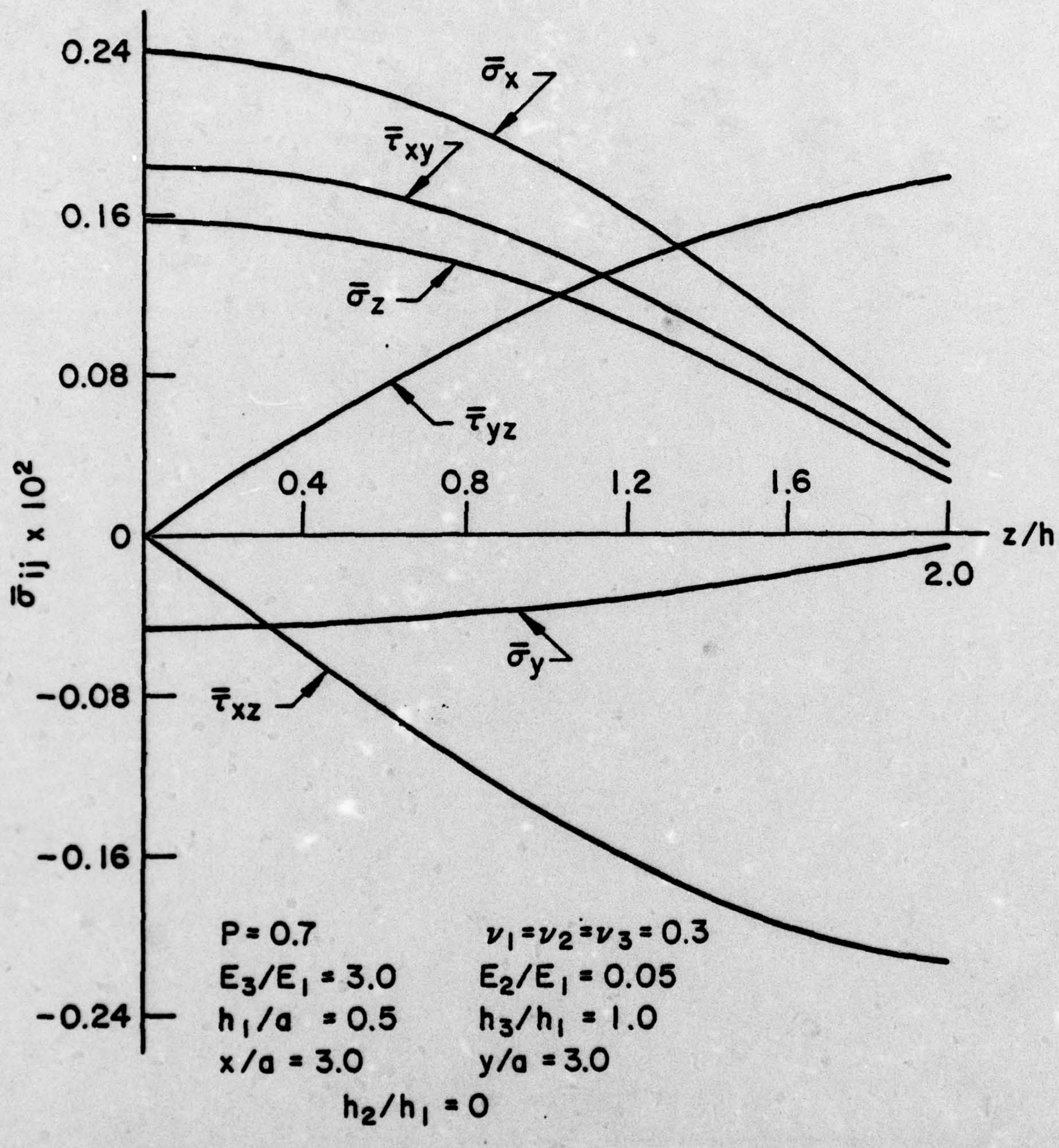

Figure 15. - Variation of stresses across the thickness for $h_2/h_1 = 0.0$ ($x/a = 3.0$, $y/a = 3.0$)

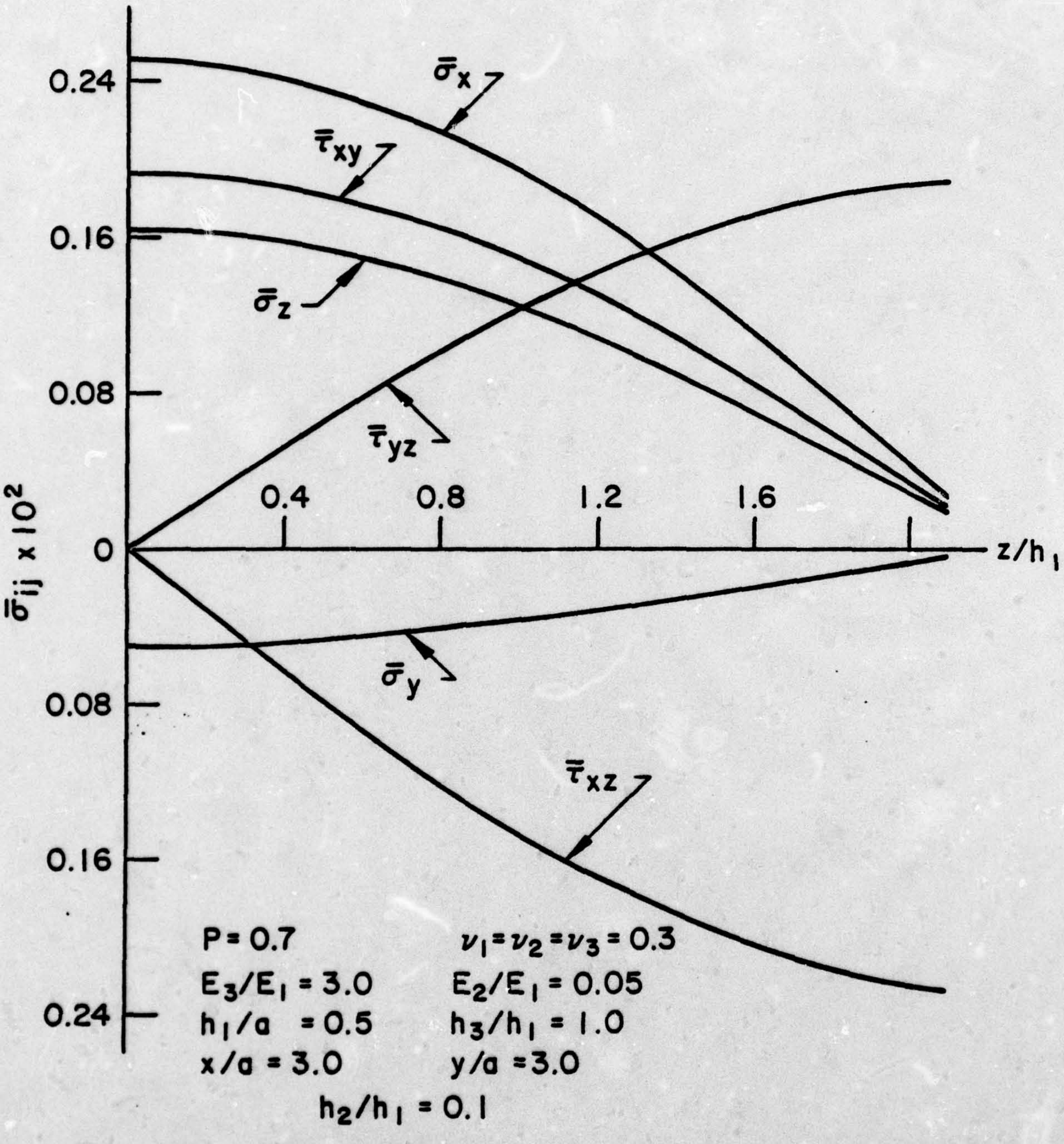

Figure 16. - Variation of stresses across the thickness for $h_2/h_1 = 0.1$ (x/a = 3.0, y/a = 3.0)

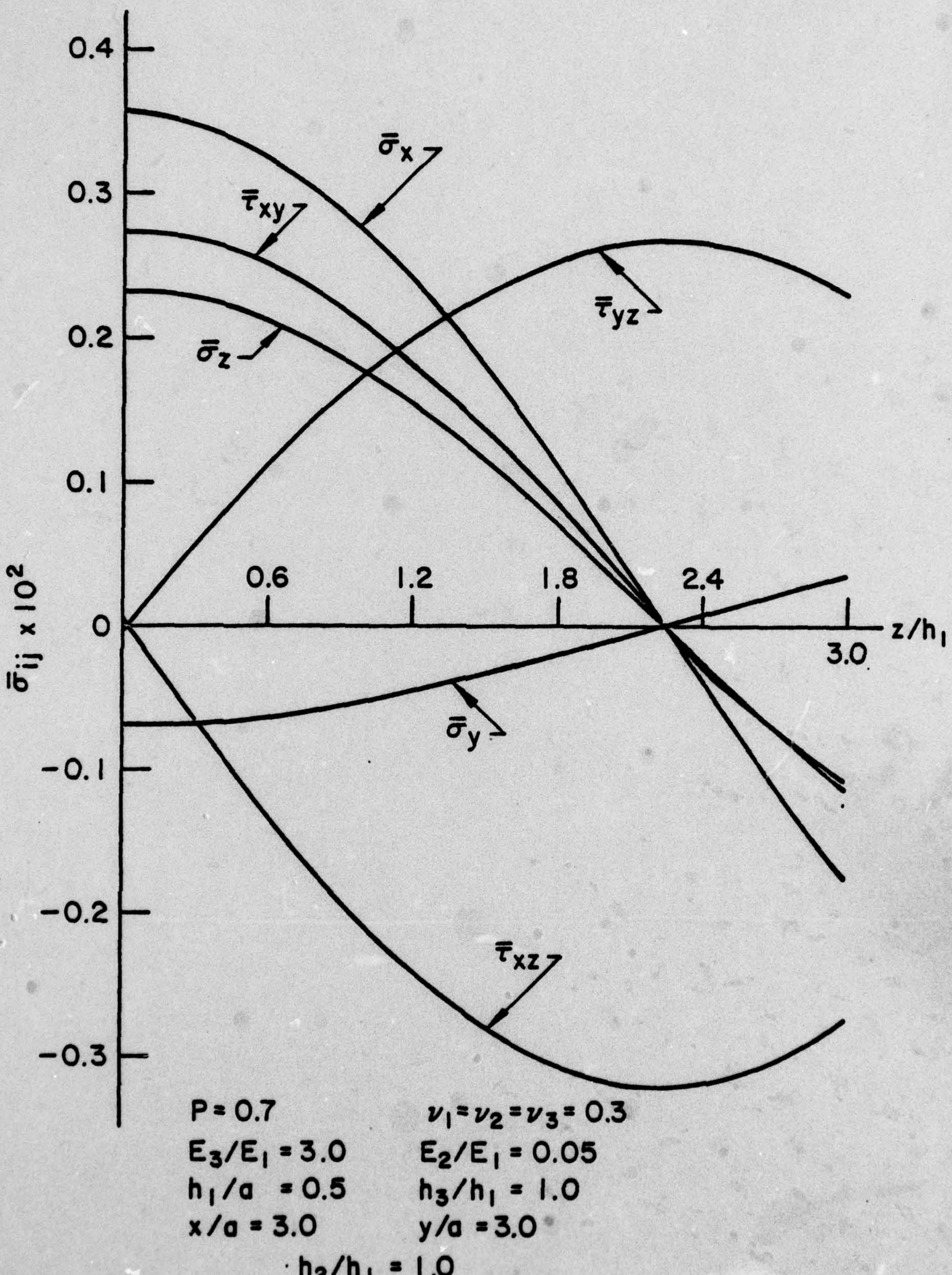

Figure 17. - Variation of stresses across the thickness for $h_2/h_1 = 1.0$ (x/a = 3.0, y/a = 3.0)

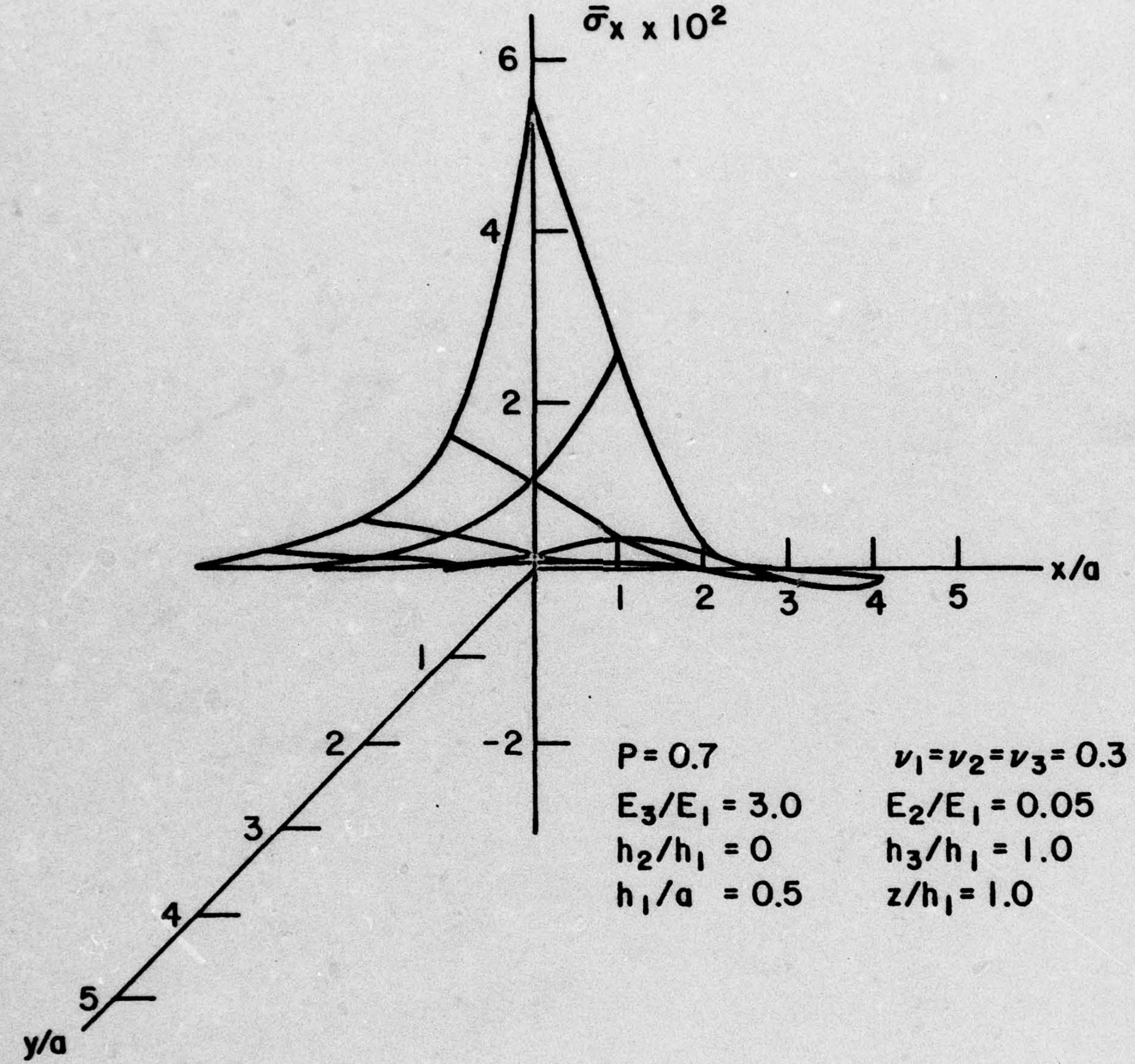

Figure 18. - Variation of $\bar{\sigma}_x$ as a function of x/a and y/a

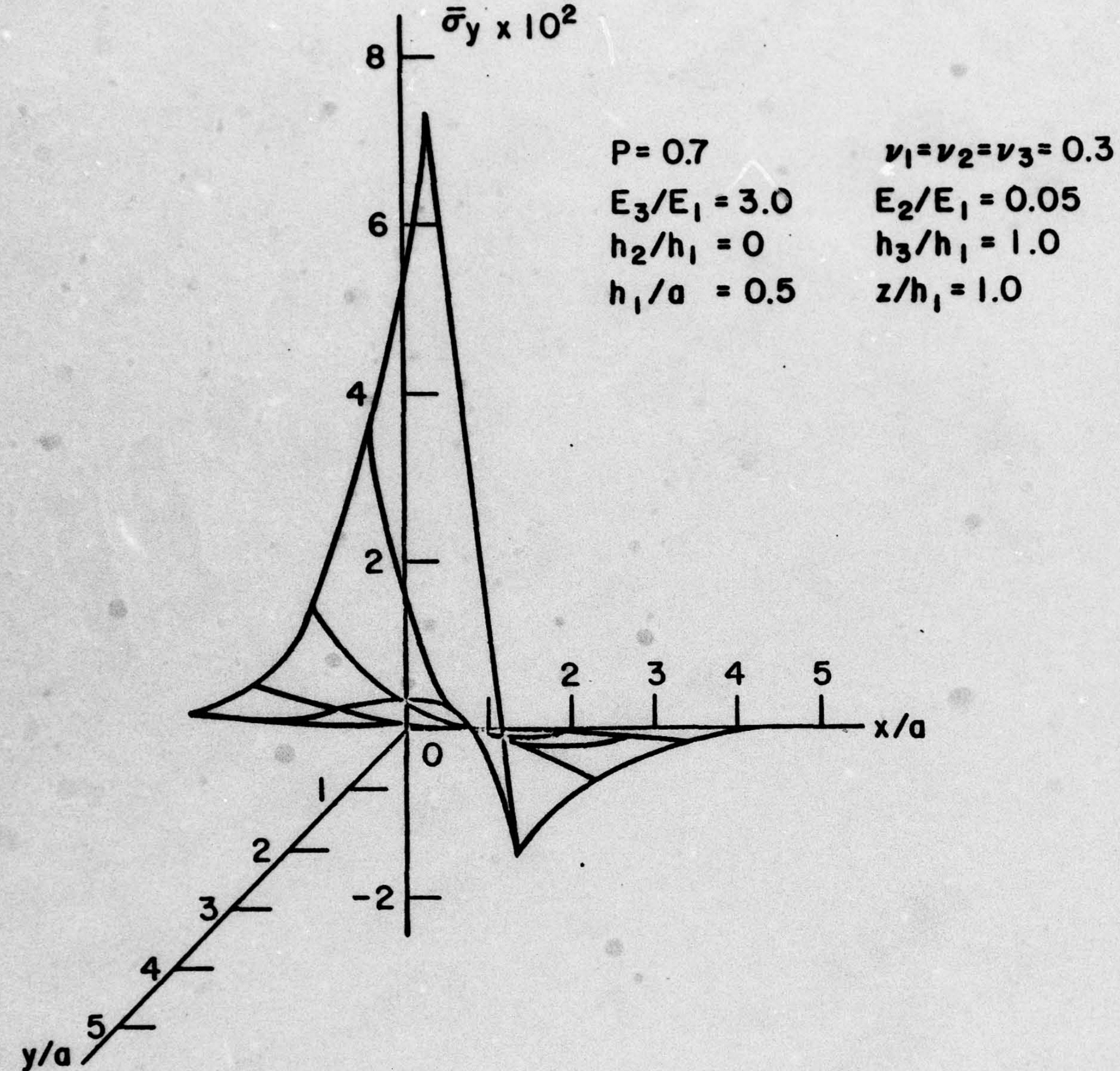

Figure 19. - Variation of $\bar{\sigma}_y$ as a function of x/a and y/a

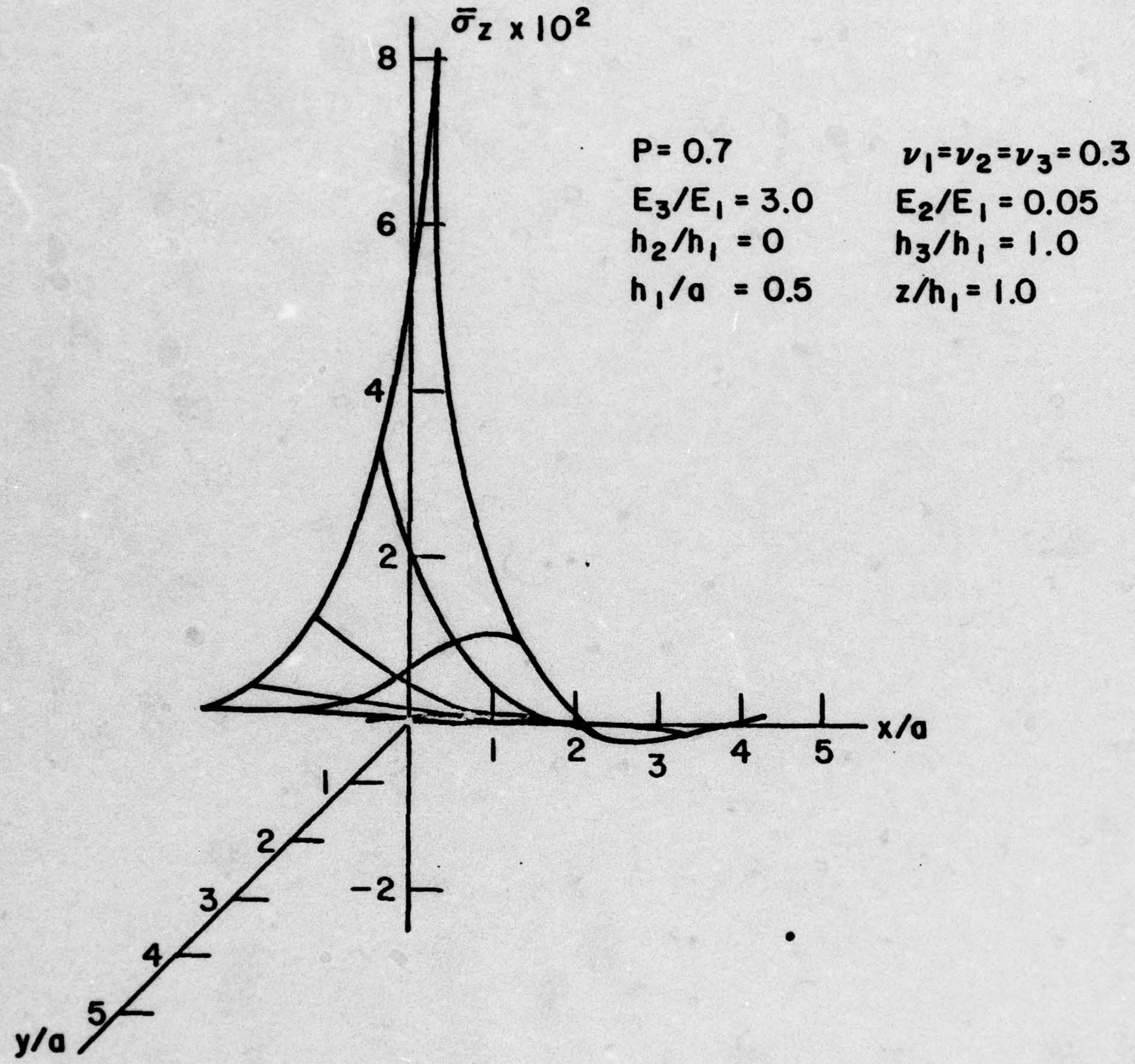

Figure 20. - Variation of $\bar{\sigma}_z$ as a function of x/a and y/a

TABLE 1

P = .700 NU = .300 H1/A = .500

E3/E1= 3.000 H3/H1= 1.00 E2/E1= .050 H2/H1= 0.00

Z/H1 = .50 X/A = 1.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	6.77899	9.05259	9.93543	-9.91788	-3.37776	.21446
2.0	2.16804	4.41560	2.36339	-4.02311	-.91718	-.12796
3.0	.79579	1.74992	.62581	-1.54993	-.24877	-.06362
4.0	.30502	.66899	.18164	-.59311	-.07292	-.02395
5.0	.11858	.25498	.05621	-.22801	-.02252	-.00858
6.0	.04661	.09802	.01782	-.08813	-.00751	-.00286
7.0	.01851	.03770	.00617	-.03357	-.00250	-.00107
8.0	.00685	.01439	.00206	-.01399	-.00071	-.00036
9.0	.00274	.00548	.00069	-.00560	-.00036	0.00000
10.0	.00069	.00206	0.00000	-.00140	0.00000	0.00000

Z/H1 = .50 X/A = 2.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	3.12971	-1.93636	1.50317	-1.27156	-.72702	1.04371
2.0	1.44970	.33175	1.24270	-1.24358	-.52543	.29274
3.0	.48803	.47707	.50860	-.67425	-.20981	.06362
4.0	.21934	.27075	.18438	-.31055	-.07578	.01287
5.0	.08705	.12681	.06512	-.13429	-.02681	.00250
6.0	.03496	.05552	.02330	-.04756	-.00929	.00036
7.0	.01371	.02330	.00823	-.02378	-.00322	0.00000
8.0	.00548	.00823	.00274	-.00979	-.00107	0.00000
9.0	.00206	.00411	.00137	-.00420	-.00036	0.00000
10.0	.00069	.00137	.00069	-.00140	0.00000	0.00000

TABLE 1 (Continued)

Z/H1 = .50 X/A = 3.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	.27623	-.80813	-.02468	.19444	-.00643	.20124
2.0	.38727	-.35574	.21249	-.10491	-.10223	.15048
3.0	.22551	-.04455	.17342	-.14688	-.07506	.06184
4.0	.10693	.03359	.08979	-.09932	-.03789	.02109
5.0	.04730	.03222	.03976	-.05316	-.01644	.00679
6.0	.02056	.01919	.01645	-.02518	-.00679	.00214
7.0	.00823	.00960	.00617	-.01119	-.00250	.00071
8.0	.00343	.00480	.00274	-.00560	-.00107	.00036
9.0	.00137	.00206	.00069	-.00280	-.00036	0.00000
10.0	.00069	.00069	.00069	-.00140	0.00000	0.00000

Z/H1 = .50 X/A = 4.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.11995	-.16451	-.08431	.09512	.05647	.00965
2.0	.00823	-.15902	-.02330	.05595	.01251	.03074
3.0	.03976	-.07814	.01851	.00560	-.00751	.02288
4.0	.03016	-.02399	.02125	-.01259	-.00894	.01144
5.0	.01714	-.00343	.01371	-.01119	-.00572	.00500
6.0	.00891	.00206	.00685	-.00839	-.00286	.00179
7.0	.00411	.00206	.00343	-.00420	-.00143	.00071
8.0	.00206	.00137	.00137	-.00280	-.00071	.00036
9.0	.00069	.00069	.00069	-.00140	-.00036	0.00000
10.0	.00069	.00069	0.00000	0.00000	0.00000	0.00000

Z/H1 = .50 X/A = 5.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.04867	.00960	-.02468	-.01399	.02359	-.00858
2.0	-.02879	-.02399	-.02399	.01679	.01465	-.00107
3.0	-.00685	-.02605	-.00960	.01259	.00536	.00322
4.0	.00206	-.01508	-.00069	.00420	.00071	.00322
5.0	.00343	-.00617	.00206	0.00000	-.00071	.00179
6.0	.00206	-.00206	.00137	-.00140	-.00071	.00107
7.0	.00137	-.00069	.00137	-.00140	-.00036	.00036
8.0	.00069	0.00000	.00069	0.00000	-.00036	0.00000
9.0	.00069	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 2

P = .700 NU = .300 H1/A = .500

E3/E1= 3.000 H3/H1= 1.00 E2/E1= .050 H2/H1= 0.00

Z/H1 = 1.00 X/A = 1.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	5.51949	7.37067	8.08948	-8.07519	-6.34596	.40292
2.0	1.76523	3.59520	1.92429	-3.27563	-1.72315	-.24041
3.0	.64794	1.42480	.50953	-1.26196	-.46738	-.11953
4.0	.24835	.54469	.14789	-.48292	-.13699	-.04499
5.0	.09655	.20761	.04576	-.18565	-.04231	-.01612
6.0	.03795	.07981	.01451	-.07175	-.01410	-.00537
7.0	.01507	.03069	.00502	-.02733	-.00470	-.00201
8.0	.00558	.01172	.00167	-.01139	-.00134	-.00067
9.0	.00223	.00446	.00056	-.00456	-.00067	0.00000
10.0	.00056	.00167	0.00000	-.00114	0.00000	0.00000

Z/H1 = 1.00 X/A = 2.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	2.54823	-1.57660	1.22389	-1.03531	-1.36589	1.96087
2.0	1.18036	.27011	1.01181	-1.01253	-.98715	.54998
3.0	.39736	.38843	.41410	-.54898	-.39419	.11953
4.0	.17859	.22044	.15013	-.25285	-.14236	.02418
5.0	.07088	.10325	.05302	-.10934	-.05036	.00470
6.0	.02846	.04521	.01897	-.03872	-.01746	.00067
7.0	.01116	.01897	.00670	-.01936	-.00604	0.00000
8.0	.00446	.00670	.00223	-.00797	-.00201	0.00000
9.0	.00167	.00335	.00112	-.00342	-.00067	0.00000
10.0	.00056	.00112	.00056	-.00114	0.00000	0.00000

TABLE 2 (Continued)

Z/H1 = 1.00 X/A = 3.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	.22491	-.65799	-.02009	.15831	-.01209	.37807
2.0	.31532	-.28965	.17301	-.08542	-.19206	.28271
3.0	.18361	-.03628	.14120	-.11959	-.14102	.11617
4.0	.08706	.02735	.07311	-.08087	-.07118	.03962
5.0	.03851	.02623	.03237	-.04328	-.03089	.01276
6.0	.01674	.01563	.01339	-.02050	-.01276	.00403
7.0	.00670	.00781	.00502	-.00911	-.00470	.00134
8.0	.00279	.00391	.00223	-.00456	-.00201	.00067
9.0	.00112	.00167	.00056	-.00228	-.00067	0.00000
10.0	.00056	.00056	.00056	-.00114	0.00000	0.00000

Z/H1 = 1.00 X/A = 4.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.09767	-.13394	-.06864	.07745	.10610	.01813
2.0	.00670	-.12948	-.01897	.04556	.02350	.05775
3.0	.03237	-.06362	.01507	.00456	-.01410	.04298
4.0	.02456	-.01953	.01730	-.01025	-.01679	.02149
5.0	.01395	-.00279	.01116	-.00911	-.01074	.00940
6.0	.00726	.00167	.00558	-.00683	-.00537	.00336
7.0	.00335	.00167	.00279	-.00342	-.00269	.00134
8.0	.00167	.00112	.00112	-.00228	-.00134	.00067
9.0	.00056	.00056	.00056	-.00114	-.00067	0.00000
10.0	.00056	.00056	0.00000	0.00000	0.00000	0.00000

Z/H1 = 1.00 X/A = 5.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.03962	.00781	-.02009	-.01139	-.04432	-.01612
2.0	-.02344	-.01953	-.01953	.01367	.02753	-.00201
3.0	-.00558	-.02121	-.00781	.01025	.01007	.00604
4.0	.00167	-.01228	-.00056	.00342	.00134	.00604
5.0	.00279	-.00502	.00167	0.00000	-.00134	.00336
6.0	.00167	-.00167	.00112	-.00114	-.00134	.00201
7.0	.00112	-.00056	.00112	-.00114	-.00067	.00067
8.0	.00056	0.00000	.00056	0.00000	-.00067	0.00000
9.0	.00056	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 3

P = .700 NU = .300 H1/A = .500

E3/E1= 3.000 H3/H1= 1.00 E2/E1= .050 H2/H1= 0.00

Z/H1 = 1.50 X/A = 1.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	3.59072	4.79501	5.26264	-5.25334	-8.54467	.54252
2.0	1.14838	2.33887	1.25185	-2.13098	-2.32017	-.32370
3.0	.42152	.92691	.33148	-.82097	-.62932	-.16095
4.0	.16156	.35435	.09621	-.31416	-.18446	-.06058
5.0	.06281	.13506	.02977	-.12078	-.05696	-.02170
6.0	.02469	.05192	.00944	-.04668	-.01899	-.00723
7.0	.00980	.01997	.00327	-.01778	-.00633	-.00271
8.0	.00363	.00762	.00109	-.00741	-.00181	-.00090
9.0	.00145	.00290	.00036	-.00296	-.00090	0.00000
10.0	.00036	.00109	0.00000	-.00074	0.00000	0.00000

Z/H1 = 1.50 X/A = 2.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	1.65776	-1.02566	.79620	-.67352	-1.83914	2.64026
2.0	.76788	.17572	.65824	-.65871	-1.32917	.74054
3.0	.25850	.25269	.26940	-.35714	-.53076	.16095
4.0	.11618	.14341	.09766	-.16449	-.19169	.03255
5.0	.04611	.06717	.03449	-.07113	-.06781	.00633
6.0	.01852	.02941	.01234	-.02519	-.02351	.00090
7.0	.00726	.01234	.00436	-.01260	-.00814	0.00000
8.0	.00290	.00436	.00145	-.00519	-.00271	0.00000
9.0	.00109	.00218	.00073	-.00222	-.00090	0.00000
10.0	.00036	.00073	.00036	-.00074	0.00000	0.00000

TABLE 3 (Continued)

Z/H1 = 1.50 X/A = 3.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	.14632	-.42805	-.01307	.10299	-.01628	.50906
2.0	.20513	-.18843	.11255	-.05557	-.25860	.38067
3.0	.11945	-.02360	.09186	-.07780	-.18988	.15643
4.0	.05664	.01779	.04756	-.05261	-.09585	.05335
5.0	.02505	.01706	.02106	-.02816	-.04159	.01718
6.0	.01089	.01017	.00871	-.01334	-.01718	.00543
7.0	.00436	.00508	.00327	-.00593	-.00633	.00181
8.0	.00182	.00254	.00145	-.00296	-.00271	.00090
9.0	.00073	.00109	.00036	-.00148	-.00090	0.00000
10.0	.00036	.00036	.00036	-.00074	0.00000	0.00000

Z/H1 = 1.50 X/A = 4.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.06354	-.08714	-.04466	.05038	.14286	.02441
2.0	.00436	-.08423	-.01234	.02964	.03165	.07776
3.0	.02106	-.04139	.00980	.00296	-.01899	.05787
4.0	.01597	-.01271	.01126	-.00667	-.02260	.02893
5.0	.00908	-.00182	.00726	-.00593	-.01447	.01266
6.0	.00472	.00109	.00363	-.00445	-.00723	.00452
7.0	.00218	.00109	.00182	-.00222	-.00362	.00181
8.0	.00109	.00073	.00073	-.00148	-.00181	.00090
9.0	.00036	.00036	.00036	-.00074	-.00090	0.00000
10.0	.00036	.00036	0.00000	0.00000	0.00000	0.00000

Z/H1 = 1.50 X/A = 5.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.02578	.00508	-.01307	-.00741	.05968	-.02170
2.0	-.01525	-.01271	-.01271	.00889	.03707	-.00271
3.0	-.00363	-.01380	-.00508	.00667	.01356	.00814
4.0	.00109	-.00799	-.00036	.00222	.00181	.00814
5.0	.00182	-.00327	.00109	0.00000	-.00181	.00452
6.0	.00109	-.00109	.00073	-.00074	-.00181	.00271
7.0	.00073	-.00036	.00073	-.00074	-.00090	.00090
8.0	.00036	0.00000	.00036	0.00000	-.00090	0.00000
9.0	.00036	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 4

P = .700 NU = .300 H1/A = .500

E3/E1= 3.000 H3/H1= 1.00 E2/E1= .050 H2/H1= .02

Z/H1 = .50 X/A = 1.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	6.84678	9.14311	10.03479	-10.01706	-3.41154	.21661
2.0	2.18972	4.45975	2.38703	-4.06334	-.92635	-.12924
3.0	.80375	1.76742	.63206	-1.56543	-.25126	-.06426
4.0	.30807	.67568	.18346	-.59905	-.07365	-.02419
5.0	.11977	.25753	.05677	-.23029	-.02274	-.00866
6.0	.04708	.09900	.01800	-.08901	-.00758	-.00289
7.0	.01869	.03808	.00623	-.03391	-.00253	-.00108
8.0	.00692	.01454	.00208	-.01413	-.00072	-.00036
9.0	.00277	.00554	.00069	-.00565	-.00036	0.00000
10.0	.00069	.00208	0.00000	-.00141	0.00000	0.00000

Z/H1 = .50 X/A = 2.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	3.16101	-1.95573	1.51820	-1.28427	-.73429	1.05415
2.0	1.46420	.33507	1.25513	-1.25602	-.53068	.29567
3.0	.49291	.48184	.51368	-.68099	-.21191	.06426
4.0	.22153	.27346	.18623	-.31365	-.07653	.01300
5.0	.08792	.12807	.06577	-.13563	-.02708	.00253
6.0	.03531	.05608	.02354	-.04804	-.00939	.00036
7.0	.01385	.02354	.00831	-.02402	-.00325	0.00000
8.0	.00554	.00831	.00277	-.00989	-.00108	0.00000
9.0	.00208	.00415	.00138	-.00424	-.00036	0.00000
10.0	.00069	.00138	.00069	-.00141	0.00000	0.00000

TABLE 4 (Continued)

Z/H1 = .50 X/A = 3.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	.27899	-.81621	-.02492	.19639	-.00650	.20325
2.0	.39115	-.35930	.21461	-.10596	-.10325	.15199
3.0	.22776	-.04500	.17515	-.14835	-.07581	.06245
4.0	.10800	.03392	.09069	-.10031	-.03827	.02130
5.0	.04777	.03254	.04015	-.05369	-.01661	.00686
6.0	.02077	.01938	.01662	-.02543	-.00686	.00217
7.0	.00831	.00969	.00623	-.01130	-.00253	.00072
8.0	.00346	.00485	.00277	-.00565	-.00108	.00036
9.0	.00138	.00208	.00069	-.00283	-.00036	0.00000
10.0	.00069	.00069	.00069	-.00141	0.00000	0.00000

Z/H1 = .50 X/A = 4.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.12115	-.16615	-.08515	.09607	.05704	.00975
2.0	.00831	-.16061	-.02354	.05651	.01264	.03105
3.0	.04015	-.07892	.01869	.00565	-.00758	.02310
4.0	.03046	-.02423	.02146	-.01272	-.00903	.01155
5.0	.01731	-.00346	.01385	-.01130	-.00578	.00505
6.0	.00900	.00208	.00692	-.00848	-.00289	.00181
7.0	.00415	.00208	.00346	-.00424	-.00144	.00072
8.0	.00208	.00138	.00138	-.00283	-.00072	.00036
9.0	.00069	.00069	.00069	-.00141	-.00036	0.00000
10.0	.00069	.00069	0.00000	0.00000	0.00000	0.00000

Z/H1 = .50 X/A = 5.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.04915	.00969	-.02492	-.01413	.02383	-.00866
2.0	-.02908	-.02423	-.02423	.01695	.01480	-.00108
3.0	-.00692	-.02631	-.00969	.01272	.00542	.00325
4.0	.00208	-.01523	-.00069	.00424	.00072	.00325
5.0	.00346	-.00623	.00208	0.00000	-.00072	.00181
6.0	.00208	-.00208	.00138	-.00141	-.00072	.00108
7.0	.00138	-.00069	.00138	-.00141	-.00036	.00036
8.0	.00069	0.00000	.00069	0.00000	-.00036	0.00000
9.0	.00069	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 5

P = .700 NU = .300 H1/A = .500

E3/E1= 3.000 H3/H1= 1.00 E2/E1= .050 H2/H1= .02

Z/H1 = 1.01 X/A = 1.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	5.54168	7.40030	8.12200	-8.10765	-6.46253	.41032
2.0	1.77233	3.60966	1.93202	-3.28880	-1.75480	-.24482
3.0	.65054	1.43053	.51158	-1.26704	-.47597	-.12173
4.0	.24935	.54688	.14849	-.48486	-.13951	-.04582
5.0	.09694	.20844	.04595	-.18640	-.04308	-.01641
6.0	.03810	.08013	.01457	-.07204	-.01436	-.00547
7.0	.01513	.03082	.00504	-.02744	-.00479	-.00205
8.0	.00560	.01177	.00168	-.01144	-.00137	-.00068
9.0	.00224	.00448	.00056	-.00457	-.00068	0.00000
10.0	.00056	.00168	0.00000	-.00114	0.00000	0.00000

Z/H1 = 1.01 X/A = 2.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	2.55847	-1.58294	1.22881	-1.03947	-1.39098	1.99689
2.0	1.18510	.27120	1.01588	-1.01660	-1.00528	.56009
3.0	.39896	.38999	.41577	-.55118	-.40143	.12173
4.0	.17931	.22133	.15073	-.25386	-.14498	.02462
5.0	.07116	.10366	.05323	-.10978	-.05129	.00479
6.0	.02858	.04539	.01905	-.03888	-.01778	.00068
7.0	.01121	.01905	.00672	-.01944	-.00615	0.00000
8.0	.00448	.00672	.00224	-.00800	-.00205	0.00000
9.0	.00168	.00336	.00112	-.00343	-.00068	0.00000
10.0	.00056	.00112	.00056	-.00114	0.00000	0.00000

TABLE 5 (Continued)

Z/H1 = 1.01 X/A = 3.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	.22581	-.66063	-.02017	.15895	-.01231	.38502
2.0	.31659	-.29081	.17370	-.08577	-.19559	.28791
3.0	.18435	-.03642	.14176	-.12007	-.14361	.11831
4.0	.08741	.02746	.07340	-.08119	-.07249	.04035
5.0	.03866	.02634	.03250	-.04345	-.03146	.01299
6.0	.01681	.01569	.01345	-.02058	-.01299	.00410
7.0	.00672	.00784	.00504	-.00915	-.00479	.00137
8.0	.00280	.00392	.00224	-.00457	-.00205	.00068
9.0	.00112	.00168	.00056	-.00229	-.00068	0.00000
10.0	.00056	.00056	.00056	-.00114	0.00000	0.00000

Z/H1 = 1.01 X/A = 4.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.09806	-.13448	-.06892	.07776	.10805	.01846
2.0	.00672	-.13000	-.01905	.04574	.02394	.05881
3.0	.03250	-.06388	.01513	.00457	-.01436	.04377
4.0	.02465	-.01961	.01737	-.01029	-.01710	.02188
5.0	.01401	-.00280	.01121	-.00915	-.01094	.00957
6.0	.00728	.00168	.00560	-.00686	-.00547	.00342
7.0	.00336	.00168	.00280	-.00343	-.00274	.00137
8.0	.00168	.00112	.00112	-.00229	-.00137	.00068
9.0	.00056	.00056	.00056	-.00114	-.00068	0.00000
10.0	.00056	.00056	0.00000	0.00000	0.00000	0.00000

Z/H1 = 1.01 X/A = 5.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.03978	.00784	-.02017	-.01144	.04514	-.01641
2.0	-.02353	-.01961	-.01961	.01372	.02804	-.00205
3.0	-.00560	-.02129	-.00784	.01029	.01026	.00615
4.0	.00168	-.01233	-.00056	.00343	.00137	.00615
5.0	.00280	-.00504	.00168	0.00000	-.00137	.00342
6.0	.00168	-.00168	.00112	-.00114	-.00137	.00205
7.0	.00112	-.00056	.00112	-.00114	-.00068	.00068
8.0	.00056	0.00000	.00056	0.00000	-.00068	0.00000
9.0	.00056	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 6

P = .700 NU = .300 H1/A = .500

E3/E1= 3.000 H3/H1= 1.00 E2/E1= .050 H2/H1= .02

Z/H1 = 1.52 X/A = 1.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	3.53777	4.72429	5.18503	-5.17587	-8.69858	.55229
2.0	1.13144	2.30438	1.23339	-2.09955	-2.36196	-.32953
3.0	.41530	.91324	.32659	-.80887	-.64066	-.16385
4.0	.15918	.34913	.09479	-.30953	-.18778	-.06167
5.0	.06188	.13307	.02933	-.11899	-.05799	-.02209
6.0	.02432	.05115	.00930	-.04599	-.01933	-.00736
7.0	.00966	.01967	.00322	-.01752	-.00644	-.00276
8.0	.00358	.00751	.00107	-.00730	-.00184	-.00092
9.0	.00143	.00286	.00036	-.00292	-.00092	0.00000
10.0	.00036	.00107	0.00000	-.00073	0.00000	0.00000

Z/H1 = 1.52 X/A = 2.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	1.63331	-1.01053	.78446	-.66359	-1.87227	2.68781
2.0	.75656	.17313	.64853	-.64899	-1.35311	.75388
3.0	.25469	.24897	.26542	-.35187	-.54032	.16385
4.0	.11447	.14130	.09622	-.16207	-.19514	.03314
5.0	.04543	.06618	.03398	-.07008	-.06904	.00644
6.0	.01824	.02897	.01216	-.02482	-.02393	.00092
7.0	.00715	.01216	.00429	-.01241	-.00828	0.00000
8.0	.00286	.00429	.00143	-.00511	-.00276	0.00000
9.0	.00107	.00215	.00072	-.00219	-.00092	0.00000
10.0	.00036	.00072	.00036	-.00073	0.00000	0.00000

TABLE 6 (Continued)

Z/H1 = 1.52 X/A = 3.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	.14416	-.42174	-.01288	.10147	-.01657	.51823
2.0	.20211	-.18565	.11089	-.05475	-.26326	.38752
3.0	.11769	-.02325	.09050	-.07665	-.19330	.15924
4.0	.05580	.01753	.04686	-.05183	-.09757	.05431
5.0	.02468	.01681	.02075	-.02774	-.04234	.01749
6.0	.01073	.01002	.00859	-.01314	-.01749	.00552
7.0	.00429	.00501	.00322	-.00584	-.00644	.00184
8.0	.00179	.00250	.00143	-.00292	-.00276	.00092
9.0	.00072	.00107	.00036	-.00146	-.00092	0.00000
10.0	.00036	.00036	.00036	-.00073	0.00000	0.00000

Z/H1 = 1.52 X/A = 4.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.06260	-.08585	-.04400	.04964	.14544	.02485
2.0	.00429	-.08299	-.01216	.02920	.03222	.07916
3.0	.02075	-.04078	.00966	.00292	-.01933	.05891
4.0	.01574	-.01252	.01109	-.00657	-.02301	.02946
5.0	.00894	-.00179	.00715	-.00584	-.01473	.01289
6.0	.00465	.00107	.00358	-.00438	-.00736	.00460
7.0	.00215	.00107	.00179	-.00219	-.00368	.00184
8.0	.00107	.00072	.00072	-.00146	-.00184	.00092
9.0	.00036	.00036	.00036	-.00073	-.00092	0.00000
10.0	.00036	.00036	0.00000	0.00000	0.00000	0.00000

Z/H1 = 1.52 X/A = 5.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.02540	.00501	-.01288	-.00730	.06075	-.02209
2.0	-.01502	-.01252	-.01252	.00876	.03774	-.00276
3.0	-.00358	-.01359	-.00501	.00657	.01381	.00828
4.0	.00107	-.00787	-.00036	.00219	.00184	.00828
5.0	.00179	-.00322	.00107	0.00000	-.00184	.00460
6.0	.00107	-.00107	.00072	-.00073	-.00184	.00276
7.0	.00072	-.00036	.00072	-.00073	-.00092	.00092
8.0	.00036	0.00000	.00036	0.00000	-.00092	0.00000
9.0	.00036	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 7

P = .700 NU = .300 H1/A = .500

E3/E1= 3.000 H3/H1= 1.00 E2/E1= .050 H2/H1= .10

Z/H1 = .50 X/A = 1.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	7.10438	9.48711	10.41233	-10.39394	-3.53990	.22476
2.0	2.27211	4.62754	2.47684	-4.21621	-.96120	-.13410
3.0	.83399	1.83392	.65584	-1.62433	-.26072	-.06668
4.0	.31966	.70110	.19036	-.62158	-.07642	-.02510
5.0	.12427	.26722	.05890	-.23896	-.02360	-.00899
6.0	.04885	.10272	.01868	-.09236	-.00787	-.00300
7.0	.01940	.03951	.00647	-.03518	-.00262	-.00112
8.0	.00718	.01509	.00216	-.01466	-.00075	-.00037
9.0	.00287	.00575	.00072	-.00586	-.00037	0.00000
10.0	.00072	.00216	0.00000	-.00147	0.00000	0.00000

Z/H1 = .50 X/A = 2.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	3.27994	-2.02931	1.57532	-1.33259	-.76192	1.09381
2.0	1.51929	.34768	1.30235	-1.30327	-.55065	.30679
3.0	.51146	.49996	.53301	-.70661	-.21989	.06668
4.0	.22987	.28374	.19323	-.32545	-.07941	.01349
5.0	.09123	.13289	.06824	-.14074	-.02809	.00262
6.0	.03664	.05819	.02442	-.04984	-.00974	.00037
7.0	.01437	.02442	.00862	-.02492	-.00337	0.00000
8.0	.00575	.00862	.00287	-.01026	-.00112	0.00000
9.0	.00216	.00431	.00144	-.00440	-.00037	0.00000
10.0	.00072	.00144	.00072	-.00147	0.00000	0.00000

TABLE 7 (Continued)

Z/H1 = .50 X/A = 3.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	.28949	-.84692	-.02586	.20377	-.00674	21090
2.0	.40586	-.37282	.22269	-.10995	-.10713	.15770
3.0	.23633	-.04669	.18174	-.15393	-.07866	.06480
4.0	.11206	.03520	.09410	-.10409	-.03971	.02210
5.0	.04957	.03376	.04166	-.05571	-.01723	.00712
6.0	.02155	.02011	.01724	-.02639	-.00712	.00225
7.0	.00862	.01006	.00647	-.01173	-.00262	.00075
8.0	.00359	.00503	.00287	-.00586	-.00112	.00037
9.0	.00144	.00216	.00072	-.00293	-.00037	0.00000
10.0	.00072	.00072	.00072	-.00147	0.00000	0.00000

Z/H1 = .50 X/A = 4.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.12571	-.17240	-.08836	.09969	.05919	.01011
2.0	.00862	-.16665	-.02442	.05864	.01311	.03221
3.0	.04166	-.08189	.01940	.00586	-.00787	.02397
4.0	.03161	-.02514	.02227	-.01319	-.00936	.01199
5.0	.01796	-.00359	.01437	-.01173	-.00599	.00524
6.0	.00934	.00216	.00718	-.00880	-.00300	.00187
7.0	.00431	.00216	.00359	-.00440	-.00150	.00075
8.0	.00216	.00144	.00144	-.00293	-.00075	.00037
9.0	.00072	.00072	.00072	-.00147	-.00037	0.00000
10.0	.00072	.00072	0.00000	0.00000	0.00000	0.00000

Z/H1 = .50 X/A = 5.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.05100	.01006	-.02586	-.01466	.02472	-.00899
2.0	-.03017	-.02514	-.02514	.01759	.01536	-.00112
3.0	-.00718	-.02730	-.01006	.01319	.00562	.00337
4.0	.00216	-.01580	-.00072	.00440	.00075	.00337
5.0	.00359	-.00647	.00216	0.00000	-.00075	00187
6.0	.00216	-.00216	.00144	-.00147	-.00075	.00112
7.0	.00144	-.00072	.00144	-.00147	-.00037	.00037
8.0	.00072	0.00000	.00072	0.00000	-.00037	0.00000
9.0	.00072	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 8

P = .700 NU = .300 H1/A = .500

E3/E1= 3.000 H3/H1= 1.00 E2/E1= .050 H2/H1= .10

Z/H1 = 1.05 X/A = 1.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	5.61039	7.49205	8.22271	-8.20818	-6.92279	.43954
2.0	1.79430	3.65441	1.95598	-3.32958	-1.87977	-.26226
3.0	.65861	1.44826	.51793	-1.28275	-.50987	-.13040
4.0	.25244	.55366	.15033	-.49087	-.14944	-.04908
5.0	.09814	.21103	.04652	-.18871	-.04615	-.01758
6.0	.03857	.08112	.01475	-.07294	-.01538	-.00586
7.0	.01532	.03120	.00511	-.02779	-.00513	-.00220
8.0	.00567	.01191	.00170	-.01158	-.00147	-.00073
9.0	.00227	.00454	.00057	-.00463	-.00073	0.00000
10.0	.00057	.00170	0.00000	-.00116	0.00000	0.00000

Z/H1 = 1.05 X/A = 2.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	2.59020	-1.60256	1.24404	-1.05236	-1.49005	2.13910
2.0	1.19980	.27456	1.02848	-1.02921	-1.07688	.59997
3.0	.40390	.39483	.42092	-.55802	-.43002	.13040
4.0	.18153	.22408	.15260	-.25701	-.15530	.02637
5.0	.07204	.10495	.05389	-.11114	-.05494	.00513
6.0	.02893	.04595	.01929	-.03936	-.01905	.00073
7.0	.01135	.01929	.00681	-.01968	-.00659	0.00000
8.0	.00454	.00681	.00227	-.00810	-.00220	0.00000
9.0	.00170	.00340	.00113	-.00347	-.00073	0.00000
10.0	.00057	.00113	.00057	-.00116	0.00000	0.00000

TABLE 8 (Continued)

Z/H1 = 1.05   X/A = 3.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	.22861	-.66882	-.02042	.16092	-.01319	.41244
2.0	.32051	-.29442	.17586	-.08683	-.20952	.30841
3.0	.18663	-.03687	.14352	-.12156	-.15384	.12673
4.0	.08850	.02780	.07431	-.08220	-.07765	.04322
5.0	.03914	.02666	.03290	-.04399	-.03370	.01392
6.0	.01702	.01588	.01361	-.02084	-.01392	.00440
7.0	.00681	.00794	.00511	-.00926	-.00513	.00147
8.0	.00284	.00397	.00227	-.00463	-.00220	.00073
9.0	.00113	.00170	.00057	-.00232	-.00073	0.00000
10.0	.00057	.00057	.00057	-.00116	0.00000	0.00000

Z/H1 = 1.05   X/A = 4.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.09927	-.13615	-.06978	.07872	.11575	.01978
2.0	.00681	-.13161	-.01929	.04631	.02564	.06300
3.0	.03290	-.05467	.01532	.00463	-.01538	.04688
4.0	.02496	-.01985	.01759	-.01042	-.01831	.02344
5.0	.01418	-.00284	.01135	-.00926	-.01172	.01026
6.0	.00737	.00170	.00567	-.00695	-.00586	.00366
7.0	.00340	.00170	.00284	-.00347	-.00293	.00147
8.0	.00170	.00113	.00113	-.00232	-.00147	.00073
9.0	.00057	.00057	.00057	-.00116	-.00073	0.00000
10.0	.00057	.00057	0.00000	0.00000	0.00000	0.00000

Z/H1 = 1.05   X/A = 5.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.04028	.00794	-.02042	-.01158	.04835	-.01758
2.0	-.02383	-.01985	-.01985	.01389	.03004	-.00220
3.0	-.00567	-.02156	-.00794	.01042	.01099	.00659
4.0	.00170	-.01248	-.00057	.00347	.00147	.00659
5.0	.00284	-.00511	.00170	0.00000	-.00147	.00366
6.0	.00170	-.00170	.00113	-.00116	-.00147	.00220
7.0	.00113	-.00057	.00113	-.00116	-.00073	.00073
8.0	.00057	0.00000	.00057	0.00000	-.00073	0.00000
9.0	.00057	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TABLE 9

P = .700 NU = .300 H1/A = .500

E3/E1= 3.000 H3/H1= 1.00 E2/E1= .050 H2/H1= .10

Z/H1 = 1.60 X/A = 1.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	3.29502	4.40014	4.82926	-4.82072	-9.29216	.58998
2.0	1.05381	2.14626	1.14876	-1.95549	-2.52314	-.35202
3.0	.38681	.85058	.30418	-.75337	-.68437	-.17503
4.0	.14826	.32517	.08829	-.28829	-.20059	-.06588
5.0	.05764	.12394	.02732	-.11083	-.06195	-.02360
6.0	.02266	.04764	.00866	-.04284	-.02065	-.00787
7.0	.00900	.01832	.00300	-.01632	-.00688	-.00295
8.0	.00333	.00700	.00100	-.00680	-.00197	-.00098
9.0	.00133	.00267	.00033	-.00272	-.00098	0.00000
10.0	.00033	.00100	0.00000	-.00068	0.00000	0.00000

Z/H1 = 1.60 X/A = 2.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	1.52124	-.94120	.73064	-.61806	-2.00003	2.87123
2.0	.70465	.16125	.60403	-.60446	-1.44545	.80532
3.0	.23721	.23188	.24721	-.32773	-.57720	.17503
4.0	.10661	.13160	.08962	-.15095	-.20346	.03540
5.0	.04231	.06164	.03165	-.06527	-.07375	.00688
6.0	.01699	.02699	.01133	-.02312	-.02557	.00098
7.0	.00666	.01133	.00400	-.01156	-.00885	0.00000
8.0	.00267	.00400	.00133	-.00476	-.00295	0.00000
9.0	.00100	.00200	.00067	-.00204	-.00098	0.00000
10.0	.00033	.00067	.00033	-.00068	0.00000	0.00000

TABLE 9 (Continued)

Z/H1 = 1.60 X/A = 3.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	.13427	-.39280	-.01199	.09451	-.01770	.55360
2.0	.18824	-.17291	.10328	-.05099	-.28122	.41397
3.0	.10961	-.02166	.08429	-.07139	-.20649	.17011
4.0	.05197	.01633	.04364	-.04828	-.10423	.05801
5.0	.02299	.01566	.01932	-.02584	-.04523	.01868
6.0	.01000	.00933	.00800	-.01224	-.01868	.00590
7.0	.00400	.00466	.00300	-.00544	-.00688	.00197
8.0	.00167	.00233	.00133	-.00272	-.00295	.00098
9.0	.00067	.00100	.00033	-.00136	-.00098	0.00000
10.0	.00033	.00033	.00033	-.00068	0.00000	0.00000

Z/H1 = 1.60 X/A = 4.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.05830	-.07996	-.04098	.04624	.15536	.02655
2.0	.00400	-.07729	-.01133	.02720	.03442	.08456
3.0	.01932	-.03798	.00900	.00272	-.02065	.06293
4.0	.01466	-.01166	.01033	-.00612	-.02458	.03147
5.0	.00833	-.00167	.00666	-.00544	-.01573	.01377
6.0	.00433	.00100	.00333	-.00408	-.00787	.00492
7.0	.00200	.00100	.00167	-.00204	-.00393	.00197
8.0	.00100	.00067	.00067	-.00136	-.00197	.00098
9.0	.00033	.00033	.00033	-.00068	-.00098	0.00000
10.0	.00033	.00033	0.00000	0.00000	0.00000	0.00000

Z/H1 = 1.60 X/A = 5.00

Y/A	SX	SY	TXY	SZ	TXZ	TYZ
1.0	-.02365	.00466	-.01199	-.00680	.06490	-.02360
2.0	-.01399	-.01166	-.01166	.00816	.04032	-.00295
3.0	-.00333	-.01266	-.00466	.00612	.01475	.00885
4.0	.00100	-.00733	-.00033	.00204	.00197	.00885
5.0	.00167	-.00300	.00100	0.00000	-.00197	.00492
6.0	.00100	-.00100	.00067	-.00068	-.00197	.00295
7.0	.00067	-.00033	.00067	-.00068	-.00098	.00098
8.0	.00033	0.00000	.00033	0.00000	-.00098	0.00000
9.0	.00033	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

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